Containers

Part One

Outline for Today

- ***Parameter Passing in C++***
  - On xeroxes and master copies.
- ***Container Types***
  - Holding lots of pieces of data.
- ***The `Vector` type***
  - Storing sequences.
- ***Recursion on `Vectors`***
  - More practice with sequences.

Parameter Passing in C++

Prediction Time!

How it Works

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int main() {
    int value = 137;
    becomeWealthy(value);
    cout << value << endl;
    return 0;
}
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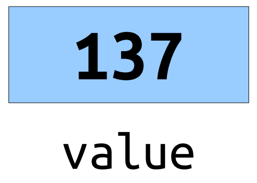

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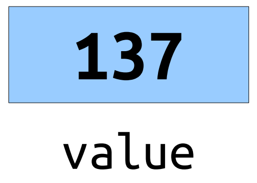

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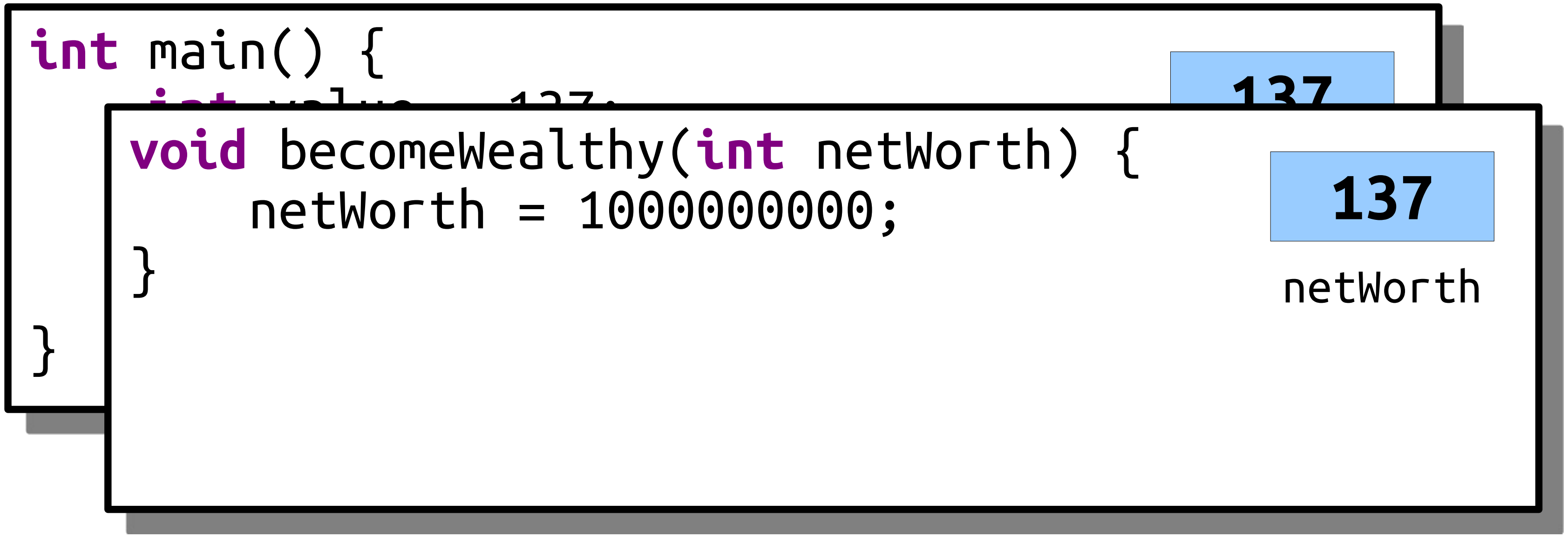
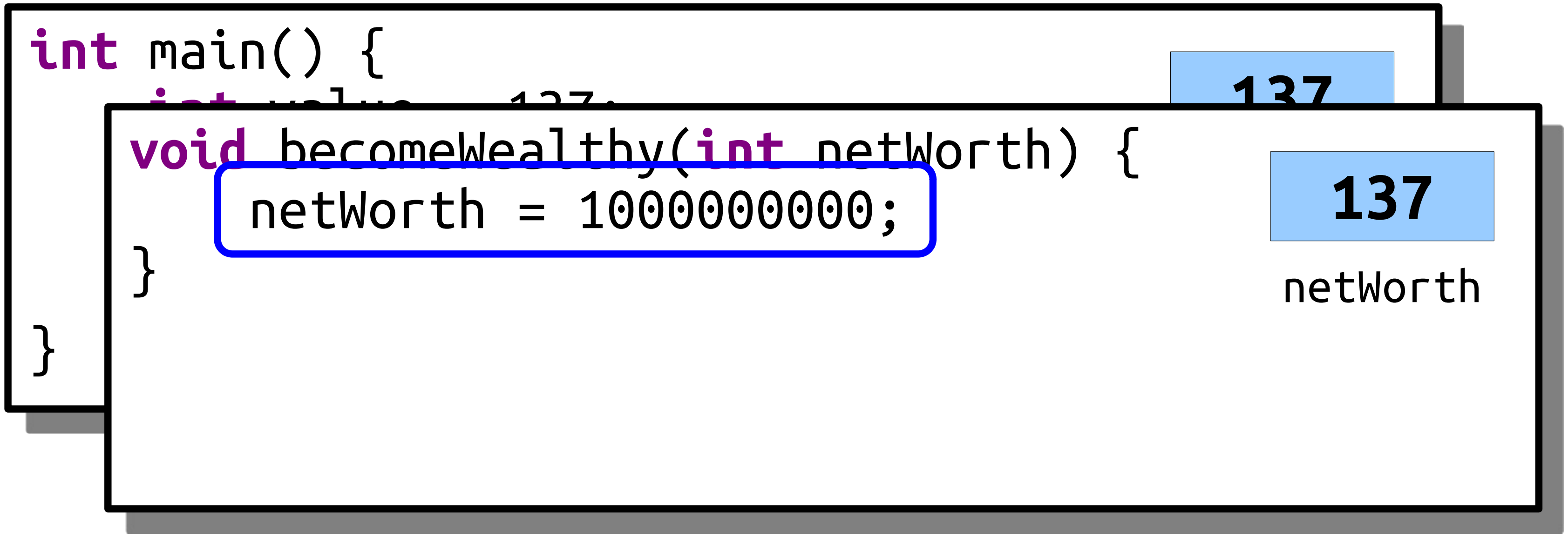


```
int main() {
}
```

```
void becomeWealthy(int netWorth) {
    netWorth = 1000000000;
}
```

How it Works



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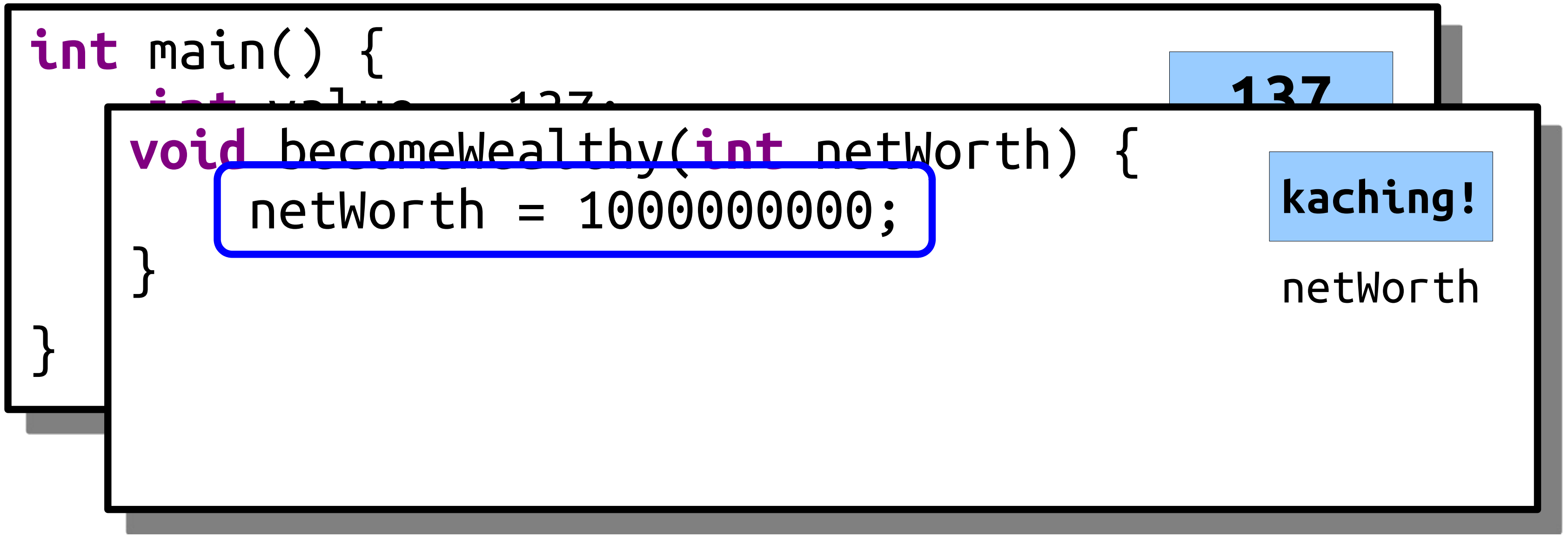

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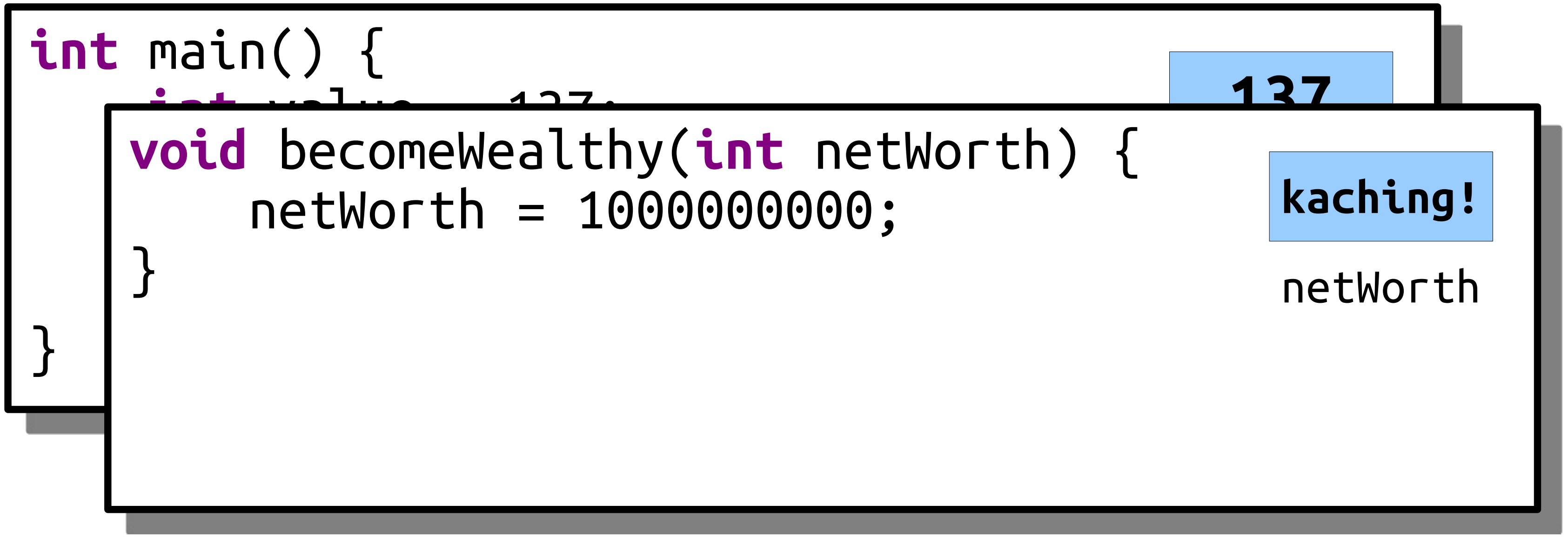

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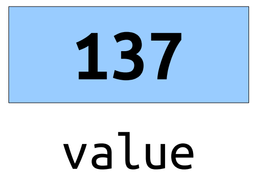

Parameter Passing in C++

- By default, in C++, parameters are passed by ***value***.

```cpp
/* This function gets a copy of the integer passed
 * into it, so we only change our local copy. The
 * caller won't see any changes.
 */
void byValue(int number) {
    number = 137;
}
```

- You can place an ampersand after the type name to take the parameter by ***reference***.

```cpp
/* This function takes its argument by reference, so
 * when the function returns, the int passed in will have
 * been permanently changed.
 */
void byReference(int& number) {
    number = 137;
}
```

How it Works Now

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    becomeWealthy(value);
    cout << value << endl;
    return 0;
}
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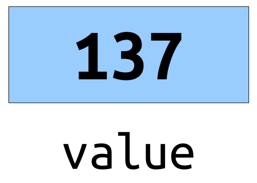

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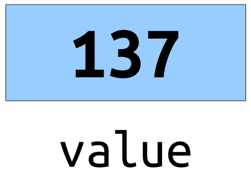

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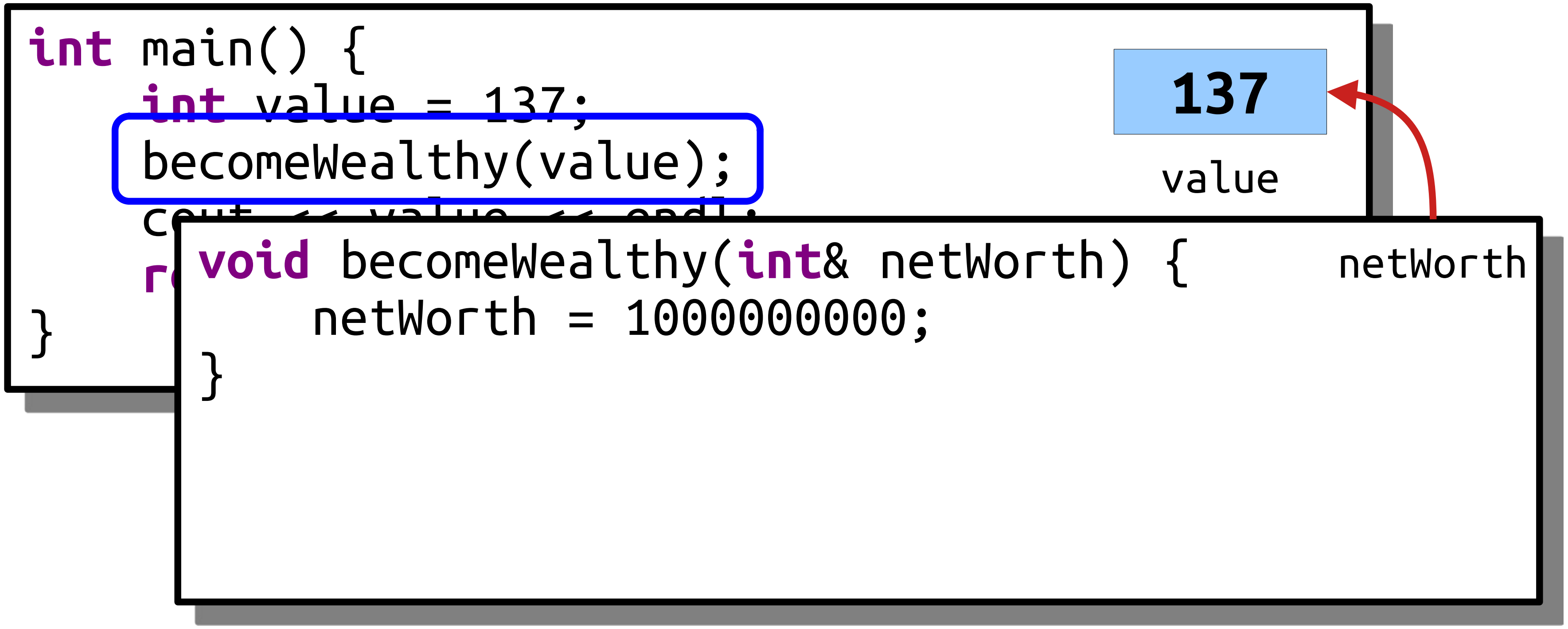

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    becomeWealthy(value);
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    r
}
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void becomeWealthy(int& netWorth) {
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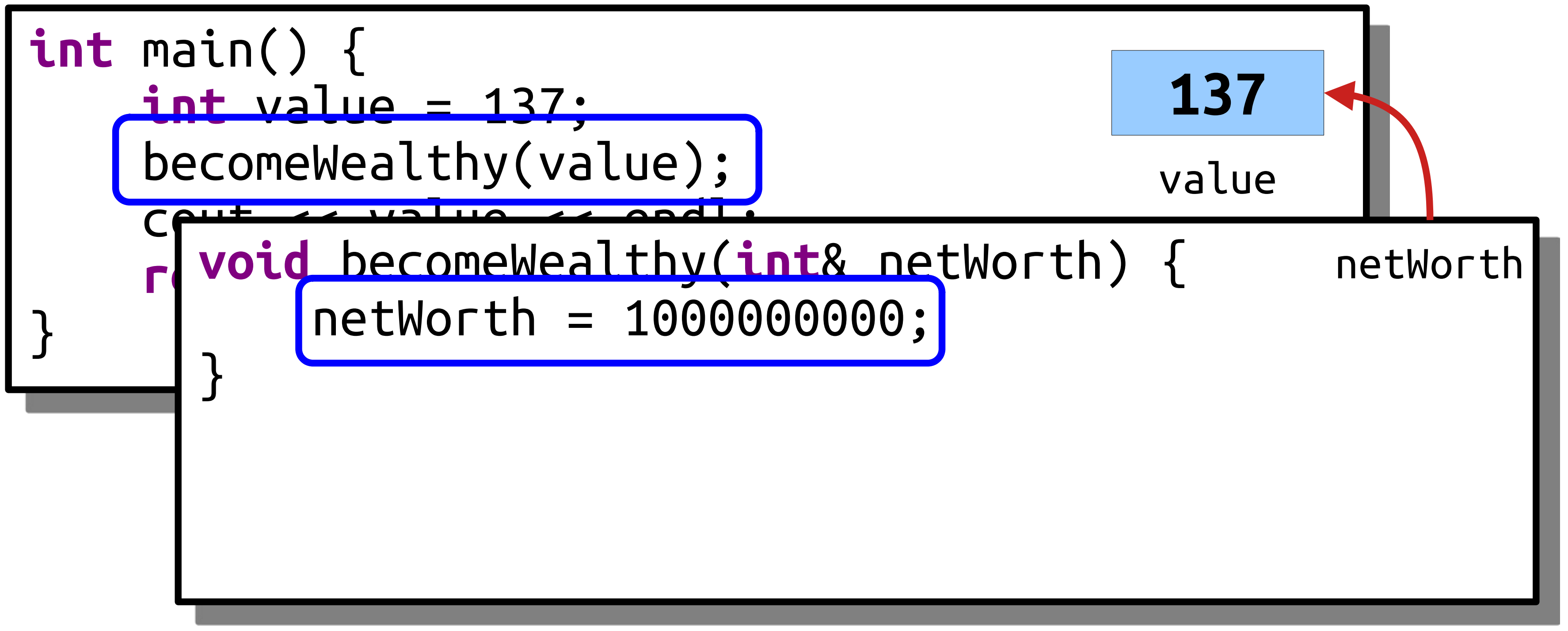

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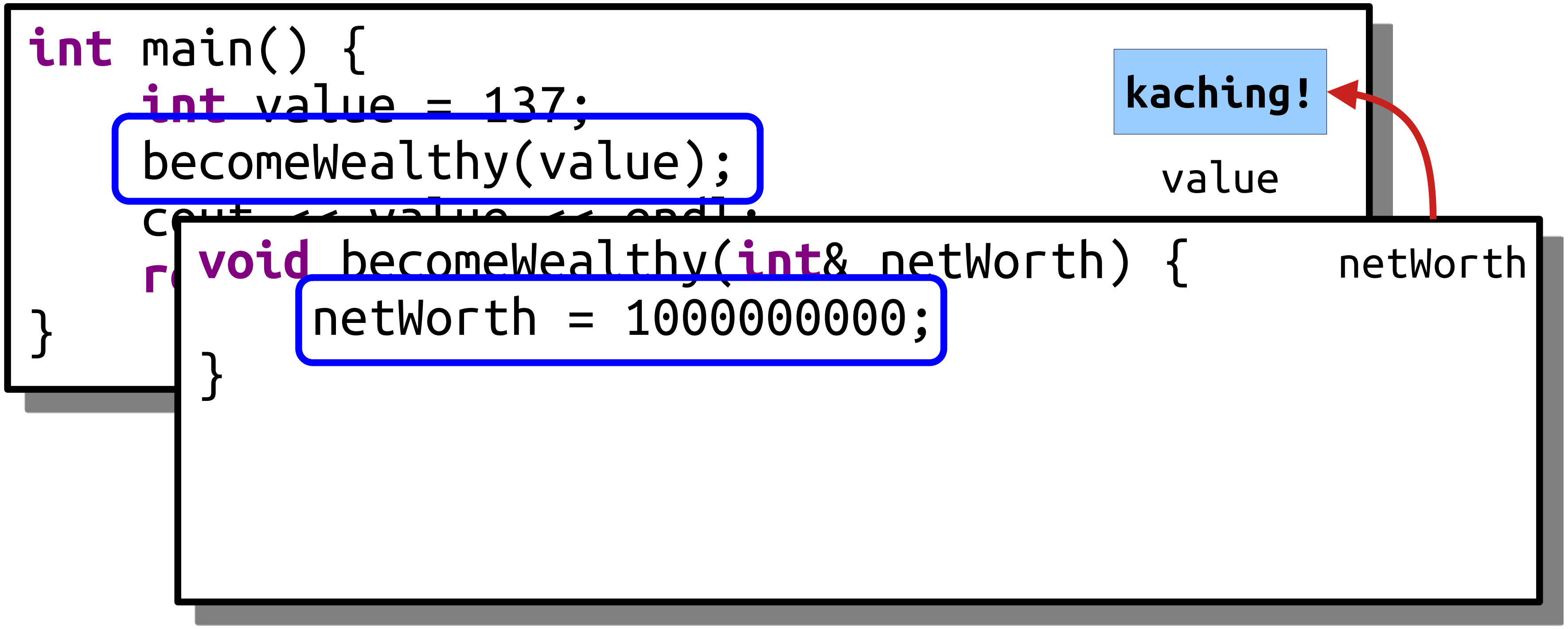

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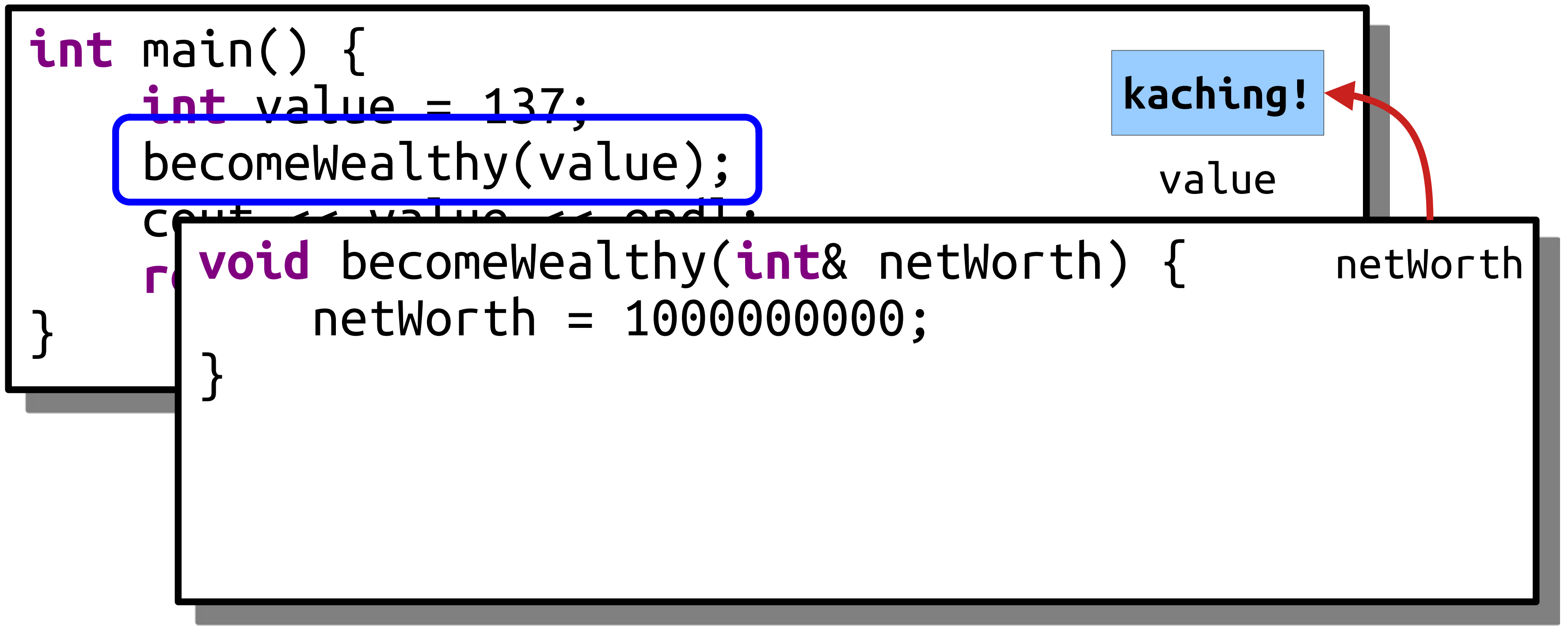

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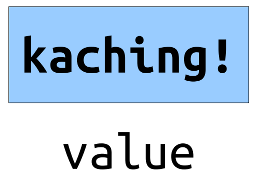
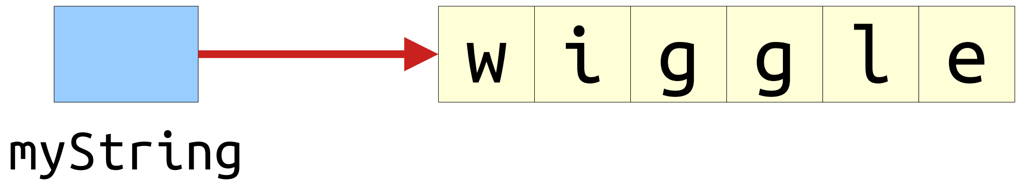


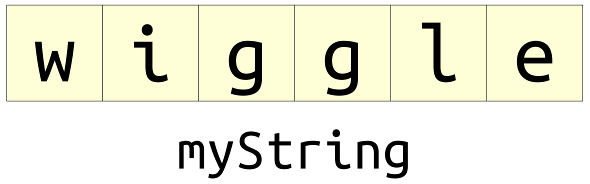
Prediction Time!

Strings in C++

- In Python, Java, and JavaScript, string variables are not the strings themselves. They’re pointers to those strings.



- In C++, a variable of type `string` is an actual, concrete, honest-to-goodness string.



How it Works

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int main() {
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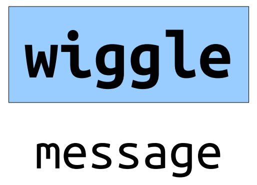

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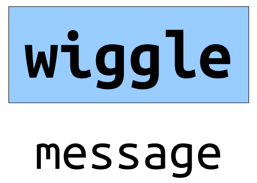

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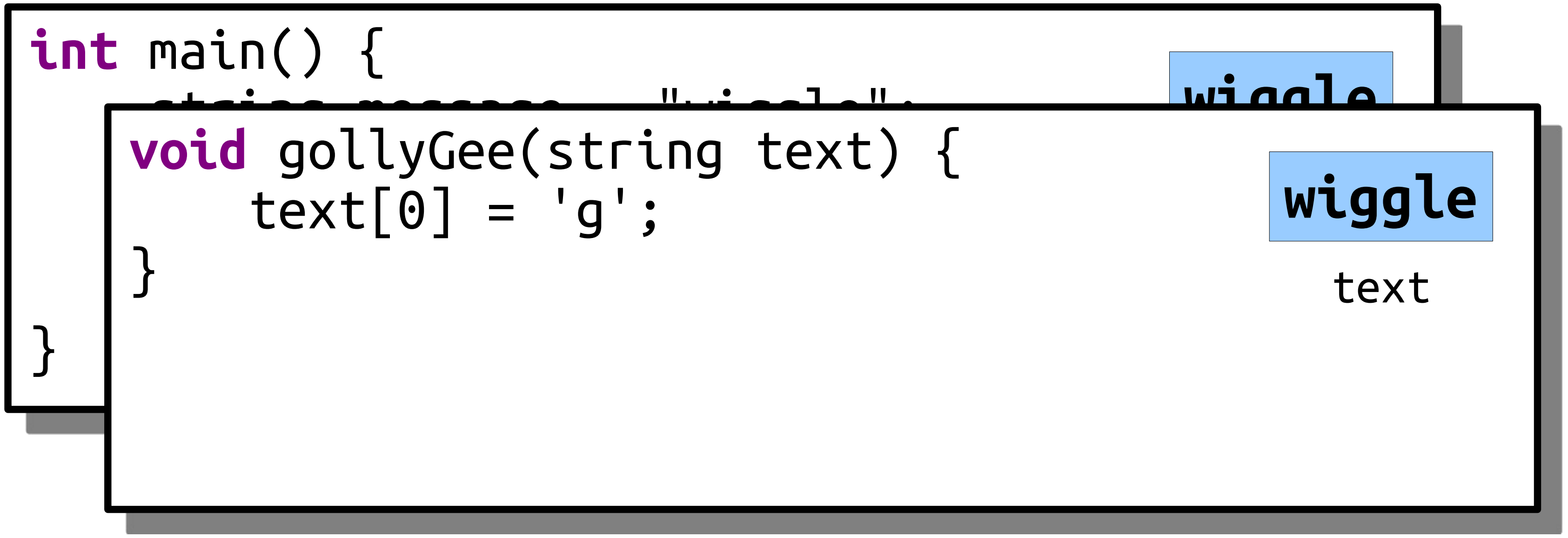

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int main() {

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void gollyGee(string text) {
    text[0] = 'g';
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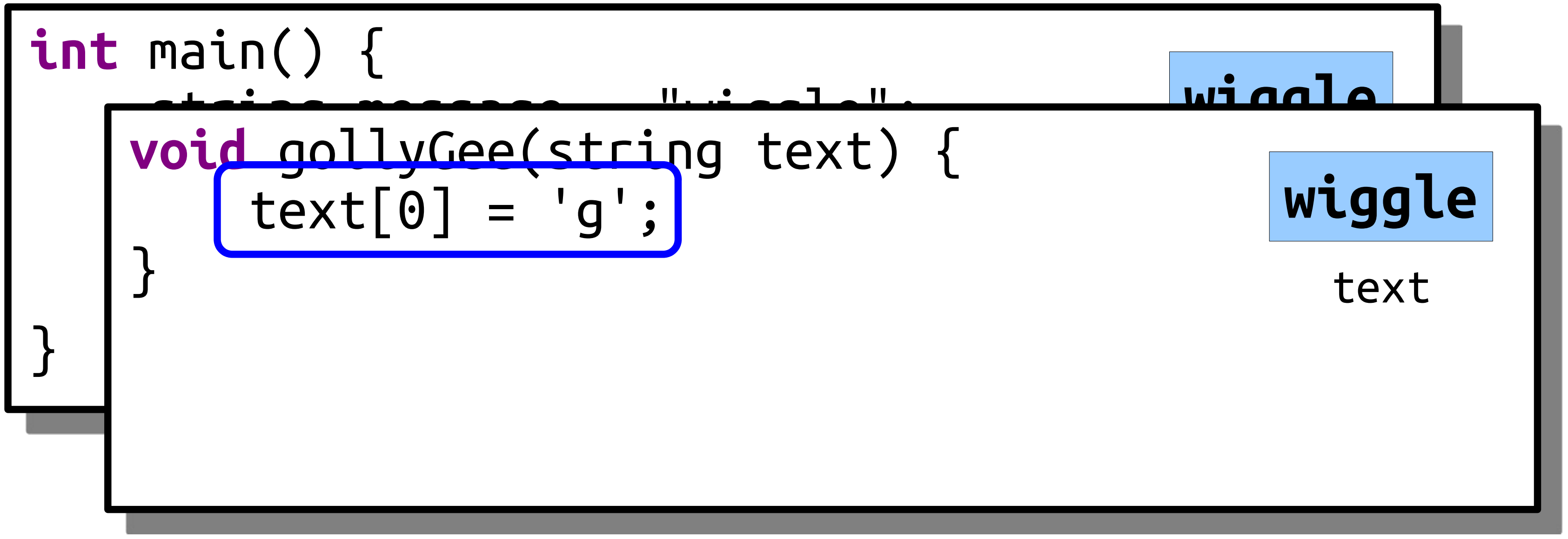

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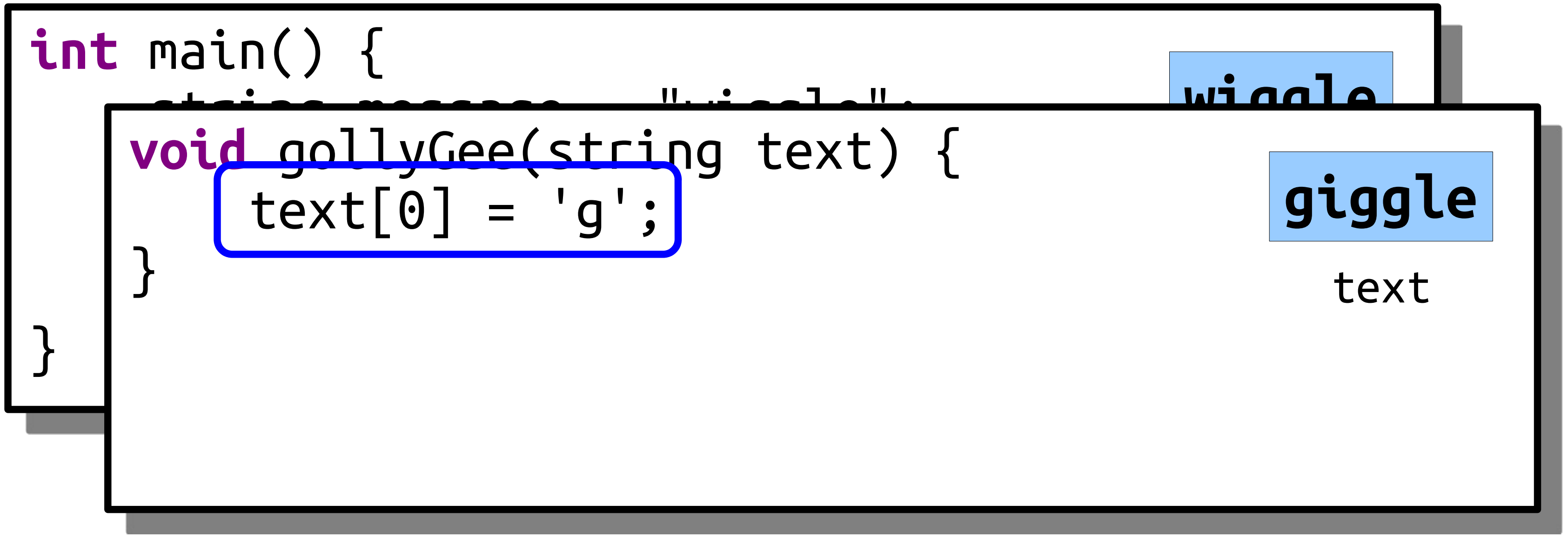

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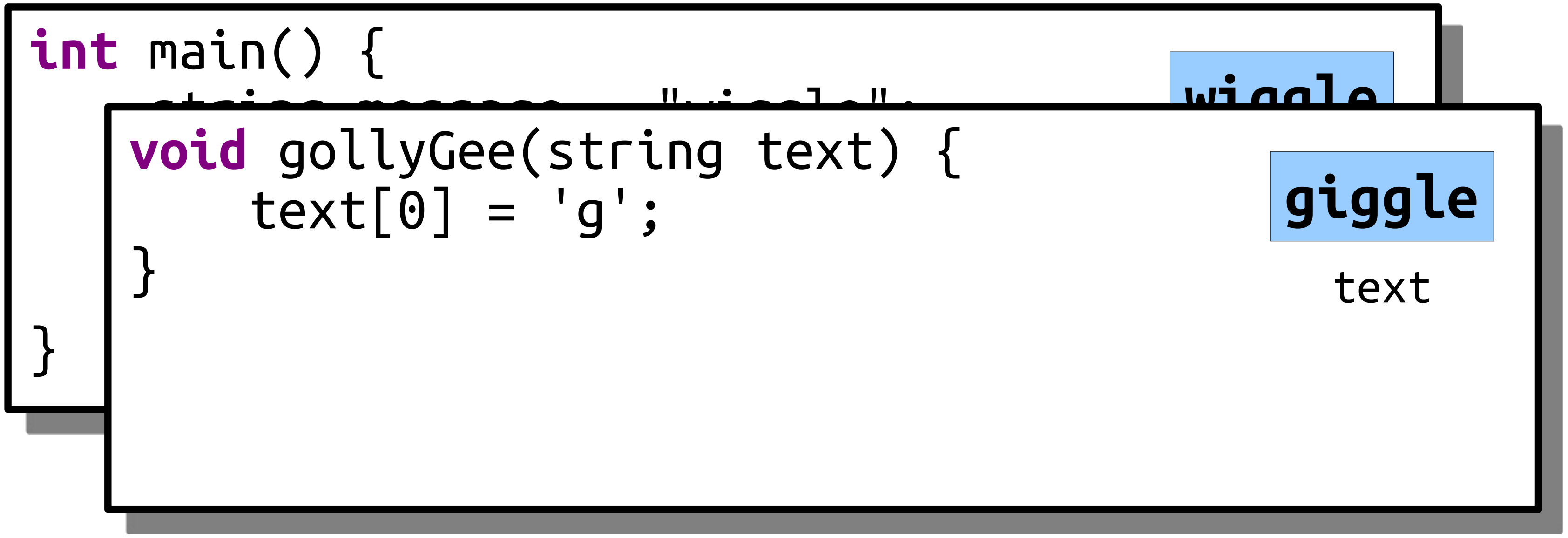

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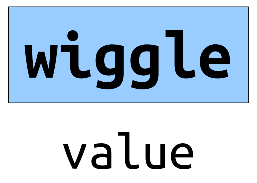

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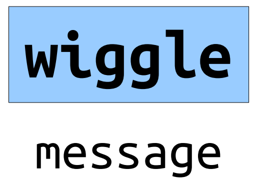

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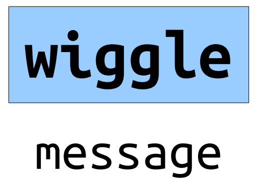

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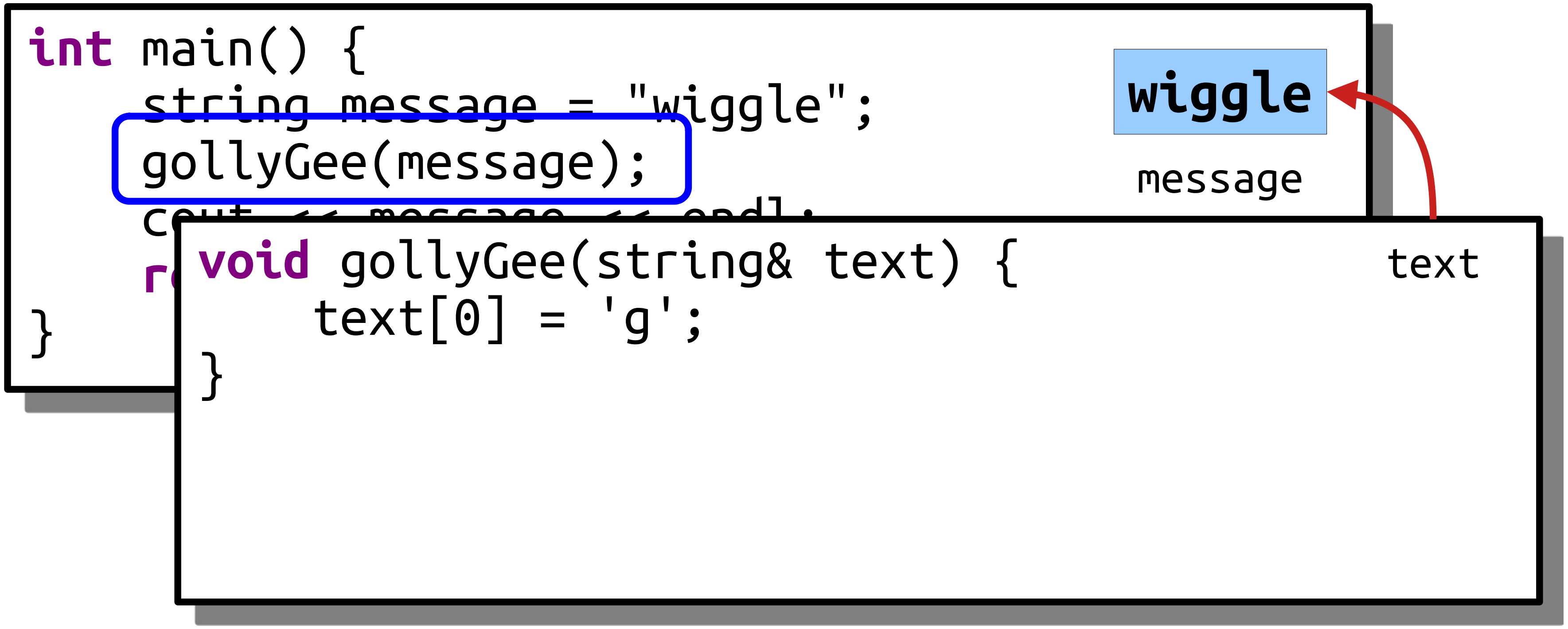

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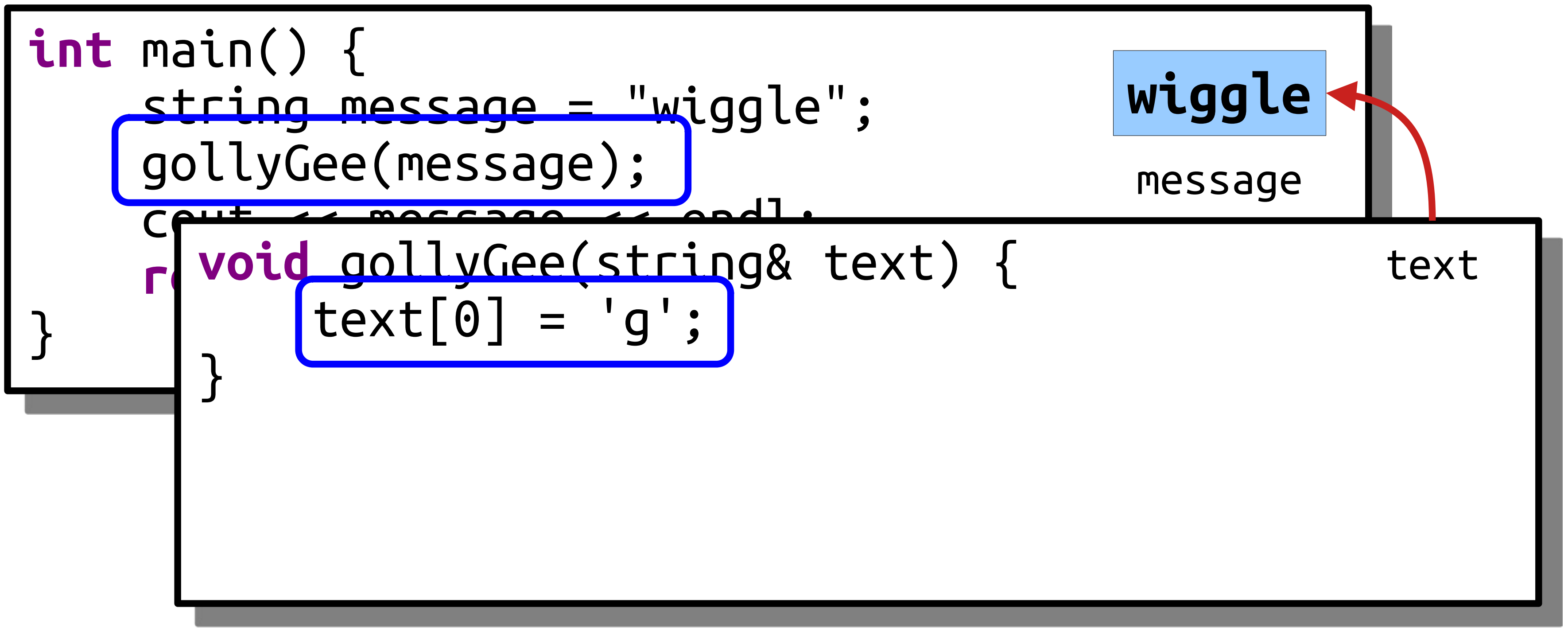

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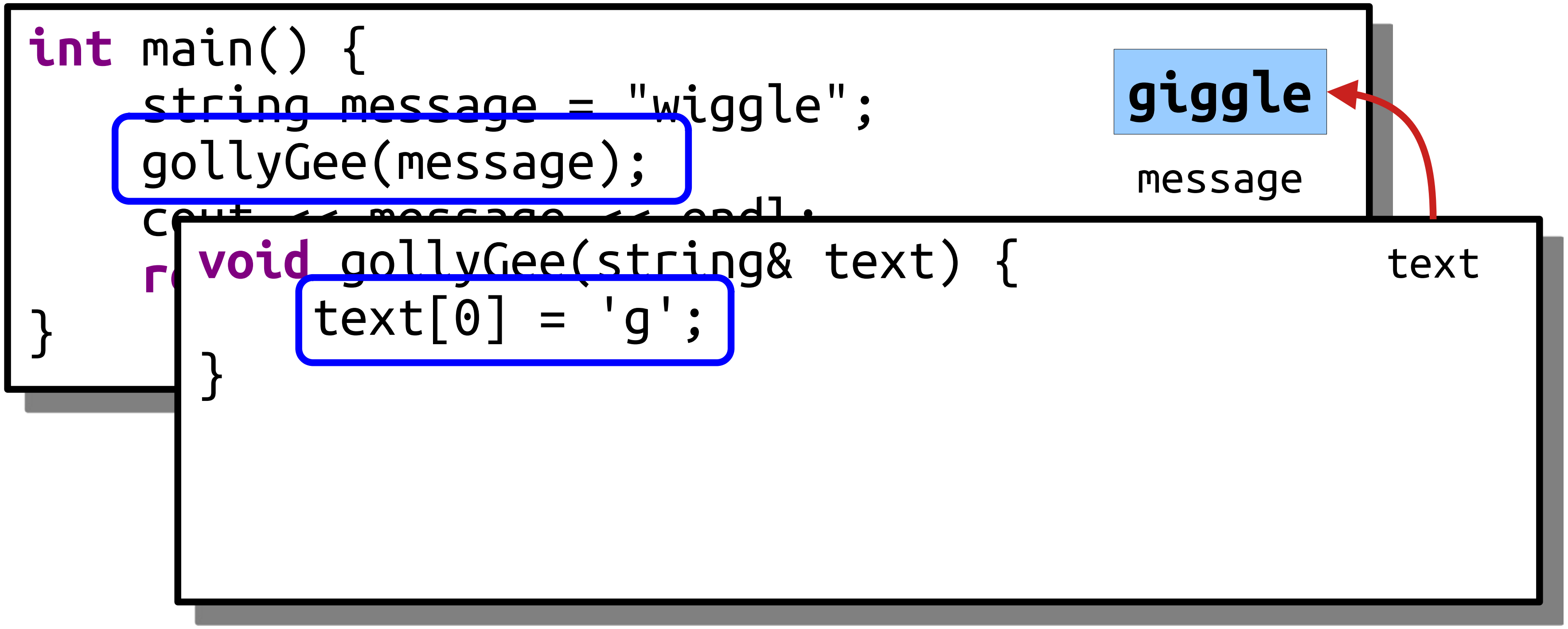

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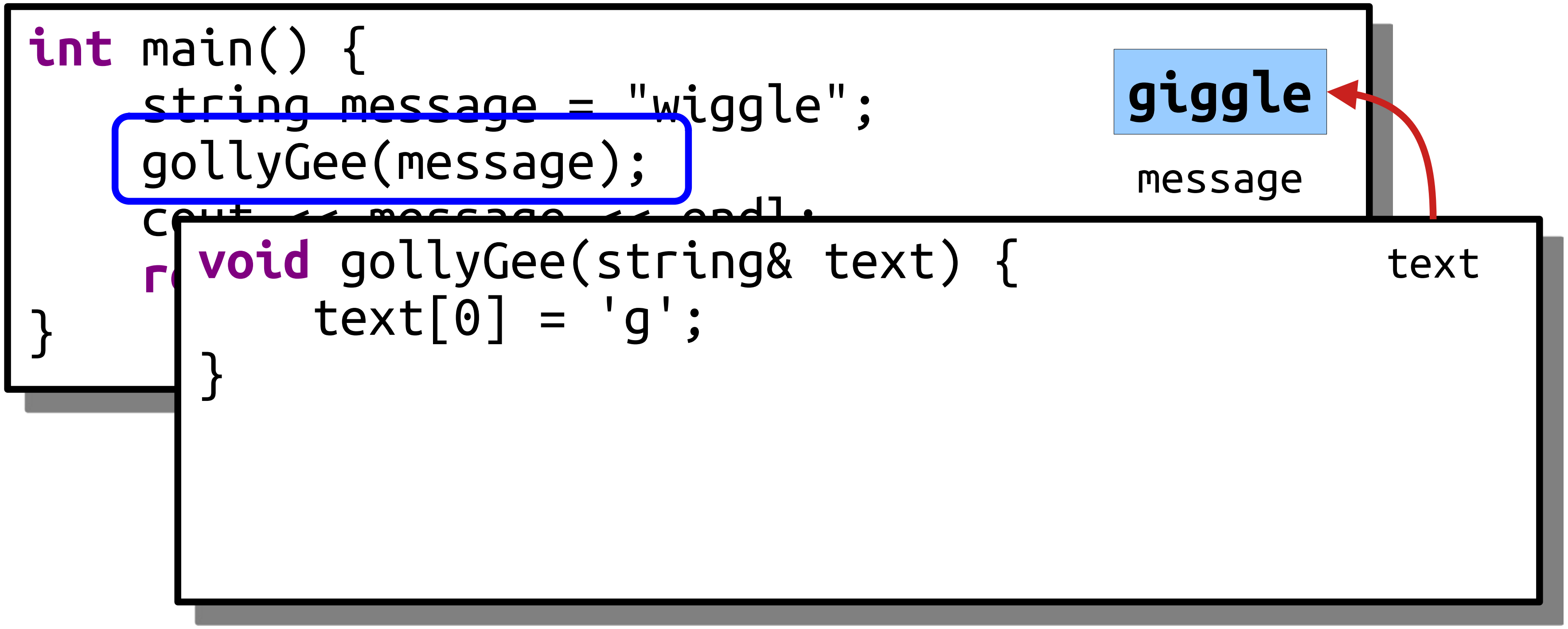

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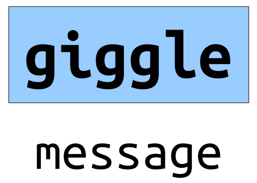

Container Types

Container Types

- A ***container type*** (also called an ***abstract data type*** or ***collection class***) is a data type used to store and organize data in some form.
  - These are things like arrays, lists, maps, dictionaries, etc.
- Our next three lectures exploring collections and how to use them appropriately.
- Later, we’ll analyze their efficiencies. For now, let’s just focus on how to use them.

Vector

Vector

- A `Vector` is a collection class representing a list of things.
- It’s similar to Java's `ArrayList`, JavaScript’s arrays, and Python’s lists.
- To make a `Vector`, use this syntax:

`Vector<`*type*`>` *name*`;`

- All elements of a `Vector` have to have the same type. You specify that type by placing it in <angle brackets> after the word `Vector`.

`Vector` in Action

```
/*       Stanford C++ Version        */
Vector<int> v = { 1, 3, 7 };


v += 271;

cout << v[0] << endl;
cout << v[v.size() - 1] << endl;

Vector<int> first = v.subList(0, 2);
Vector<int> last  = v.subList(2);

v.remove(0);
```

```
"""    Python Version   """
v = [1, 3, 7]


v.append(271)

print(v[0])
print(v[-1])

first = v[0:2]
last  = v[2:]

del v[0]
```

```
/*           Java Version            */
List<> v = new ArrayList<Integer>();
v.add(1); v.add(3); v.add(7);

v.add(271);

System.out.println(v.get(0));
System.out.println(v.get(v.size()-1));

List<Integer> first = v.subList(0, 2);
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v.remove(0);
```

```
//    JavaScript Version
let v = [1, 3, 7];


v.push(271);

console.log(v[0]);
console.log(v[v.length - 1]);

let first = v.slice(0, 2);
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v.splice(0, 0);
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Note the use of curly braces rather than square brackets here.

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We append elements using the **+=** operator.

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We select individual elements out of a **Vector** using square brackets. Everything is zero-indexed.

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v.add(1); v.add(3); v.add(7);

v.add(271);

System.out.println(v.get(0));
System.out.println(v.get(v.size()-1));

List<Integer> first = v.subList(0, 2);
List<Integer> last  = v.subList(2);

v.remove(0);
```

```
let last  = v.slice(2);

v.splice(0, 0);
```

C++ doesn't support negative array indices to mean "count from the back." We have to do some math to find the index of the last element.

We use the syntax **`v.size()`** to get the length of a **`Vector`**.

```
/*       Stanford C++ Version       */
Vector<int> v = { 1, 3, 7 };


v += 271;

cout << v[0] << endl;
cout << v[v.size() - 1] << endl;

Vector<int> first = v.subList(0, 2);
Vector<int> last  = v.subList(2);

v.remove(0);
```

```
"""    Python Version   """
v = [1, 3, 7]


v.append(271)

print(v[0])
print(v[-1])

first = v[0:2]
last  = v[2:]

del v[0]
```

```
/*          Java Version            */
List<> v = new ArrayList<Integer>();
v.add(1); v.add(3); v.add(7);

v.add(271);

System.out.println(v.get(0));
System.out.println(v.get(v.size()-1));

List<Integer> first = v.subList(0, 2);
List<Integer> last  = v.subList(2);

v.remove(0);
```

```
//    JavaScript Version

v.splice(0, 0);
```

The **subList** member function is used to get a subrange of the **subList**. Here, first will be the first two elements of the **Vector**, and last will be the list starting at position 2.

```
/*       Stanford C++ Version       */
Vector<int> v = { 1, 3, 7 };


v += 271;

cout << v[0] << endl;
cout << v[v.size() - 1] << endl;

Vector<int> first = v.subList(0, 2);
Vector<int> last  = v.subList(2);

v.remove(0);
```

```
"""    Python Version   """
v = [1, 3, 7]


v.append(271)

print(v[0])
print(v[-1])

first = v[0:2]
last  = v[2:]

del v[0]
```

```
/*          Java Version          */
List<> v = new ArrayList<Integer>();
v.add(1); v.add(3); v.add(7);

v.add(271);

System.out.println(v.get(0));
System.out.println(v.get(v.size()-1));

List<Integer> first = v.subList(0, 2);
List<Integer> last  = v.subList(2);

v.remove(0);
```

```
//     JavaScript Version

console.log(v[v.length - 1]);

let first = v.slice(0, 2);
let last  = v.slice(2);

v.splice(0, 0);
```

We can use the `remove` member function to remove the element at a given index.

```
/*      Stanford C++ Version      */
Vector<int> v = { 1, 3, 7 };


v += 271;

cout << v[0] << endl;
cout << v[v.size() - 1] << endl;

Vector<int> first = v.subList(0, 2);
Vector<int> last  = v.subList(2);

v.remove(0);
```

```
"""    Python Version   """
v = [1, 3, 7]


v.append(271)

print(v[0])
print(v[-1])

first = v[0:2]
last  = v[2:]

del v[0]
```

```
/*          Java Version          */
List<> v = new ArrayList<Integer>();
v.add(1); v.add(3); v.add(7);

v.add(271);

System.out.println(v.get(0));
System.out.println(v.get(v.size()-1));

List<Integer> first = v.subList(0, 2);
List<Integer> last  = v.subList(2);

v.remove(0);
```

```
//    JavaScript Version
let v = [1, 3, 7];


v.push(271);

console.log(v[0]);
console.log(v[v.length - 1]);

let first = v.slice(0, 2);
let last  = v.slice(2);

v.splice(0, 0);
```

```
/*      Stanford C++ Version         */
Vector<string> v = { "A", "B", "C" };


/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    cout << v[i] << endl;
}

/* Range-based for loop. */
for (string elem: v) {
    cout << elem << endl;
}
```

```
"""     Python Version     """
v = ["A", "B", "C"]


# Counting for loop.
for i in range(len(v)):
    print(v[i])


# Range-based for loop.
for elem in v:
    print(elem)
```

```
/*           Java Version            */
List<> v = new ArrayList<String>();
v.add("A"); v.add("B"); v.add("C");

/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    System.out.println(v[i]);
}

/* Range-based for loop. */
for (String elem: v) {
    System.out.println(elem);
}
```

```
//    JavaScript Version
let v = ["A", "B", "C"];


// Counting for loop.
for (let i in v) {
    console.log(v[i]);
}

// Range-based for loop.
for (let elem of v) {
    console.log(elem);
}
```

```
/*      Stanford C++ Version         */
Vector<string> v = { "A", "B", "C" };


/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    cout << v[i] << endl;
}

/* Range-based for loop. */
for (string elem: v) {
    cout << elem << endl;
}
```

```
"""     Python Version       """
v = ["A", "B", "C"]


# Counting for loop.
for i in range(len(v)):
    print(v[i])


# Range-based for loop.
for elem in v:
    print(elem)
```

```
/*          Java Version             */
List<> v = new ArrayList<String>();
v.add("A"); v.add("B"); v.add("C");

/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    System.out.println(v[i]);
}

/* Range-based for loop. */
for (String elem: v) {
    System.out.println(elem);
}
```

```
//    JavaScript Version
let v = ["A", "B", "C"];


// Counting for loop.
for (let i in v) {
    console.log(v[i]);
}

// Range-based for loop.
for (let elem of v) {
    console.log(elem);
}
```

```
/*       Stanford C++ Version          */
Vector<string> v = { "A", "B", "C" };


/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    cout << v[i] << endl;
}

/* Range-based for loop. */
for (string elem: v) {
    cout << elem << endl;
}
```

```
"""      Python Version     """
v = ["A", "B", "C"]


# Counting for loop.
for i in range(len(v)):
    print(v[i])


# Range-based for loop.
for elem in v:
    print(elem)
```

```
/*            Java Version             */
List<> v = new ArrayList<String>();
v.add("A"); v.add("B"); v.add("C");

/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    System.out.println(v[i]);
}

/* Range-based for loop. */
for (String elem: v) {
    System.out.println(elem);
}
```

We can iterate over the elements of a **Vector** by counting upward from 0 (inclusive) to its size (exclusive) and accessing each element.

```
    console.log(elem);
}
```

```
/*      Stanford C++ Version        */
Vector<string> v = { "A", "B", "C" };


/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    cout << v[i] << endl;
}

/* Range-based for loop. */
for (string elem: v) {
    cout << elem << endl;
}
```

```
"""      Python Version      """
v = ["A", "B", "C"]


# Counting for loop.
for i in range(len(v)):
    print(v[i])


# Range-based for loop.
for elem in v:
    print(elem)
```

```
/*            Java Version          */
List<> v = new ArrayList<String>();
v.add("A"); v.add("B"); v.add("C");

/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    System.out.println(v[i]);
}

/* Range-based for loop. */
for (String elem: v) {
    System.out.println(elem);
}
```

We can also use this loop structure, which visits each element of the `Vector` in the order in which they appear.

```
}

// Range-based for loop.
for (let elem of v) {
    console.log(elem);
}
```

```cpp
/*      Stanford C++ Version          */
Vector<string> v = { "A", "B", "C" };


/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    cout << v[i] << endl;
}

/* Range-based for loop. */
for (string elem: v) {
    cout << elem << endl;
}
```

```python
"""     Python Version     """
v = ["A", "B", "C"]


# Counting for loop.
for i in range(len(v)):
    print(v[i])


# Range-based for loop.
for elem in v:
    print(elem)
```

```java
/*          Java Version          */
List<> v = new ArrayList<String>();
v.add("A"); v.add("B"); v.add("C");

/* Counting for loop. */
for (int i = 0; i < v.size(); i++) {
    System.out.println(v[i]);
}

/* Range-based for loop. */
for (String elem: v) {
    System.out.println(elem);
}
```

```javascript
//     JavaScript Version
let v = ["A", "B", "C"];


// Counting for loop.
for (let i in v) {
    console.log(v[i]);
}

// Range-based for loop.
for (let elem of v) {
    console.log(elem);
}
```

To read more about the `Vector` and how to use it, check out the

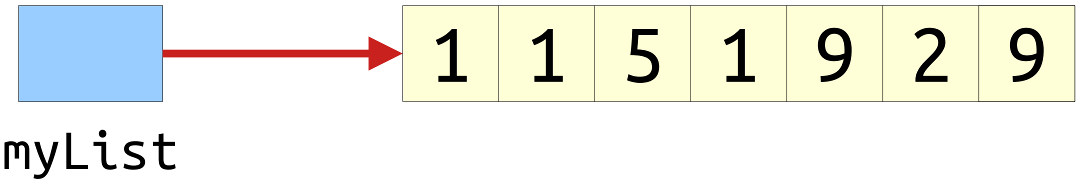
Stanford C++ Library Documentation

up on the course website.

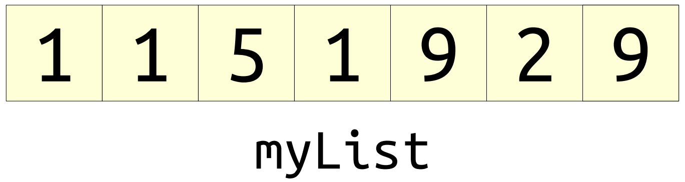
Prediction Time!

Objects in C++

- In Python, Java, and JavaScript, object variables are not the objects themselves. They're pointers to those objects:



- In C++, a variable of object type is an actual, concrete, honest-to-goodness object.



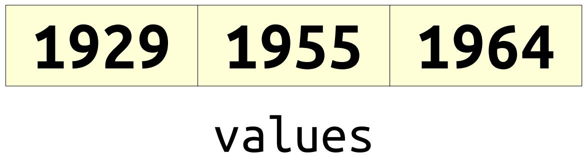
How it Works

```
int main() {
    Vector<int> values = { 1929, 1955, 1964 };
    dream(values);
    cout << values << endl;
    return 0;
}
```

How it Works

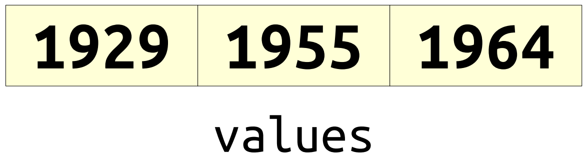
```
int main() {
    Vector<int> values = { 1929, 1955, 1964 };
    dream(values);
    cout << values << endl;
    return 0;
}
```

How it Works



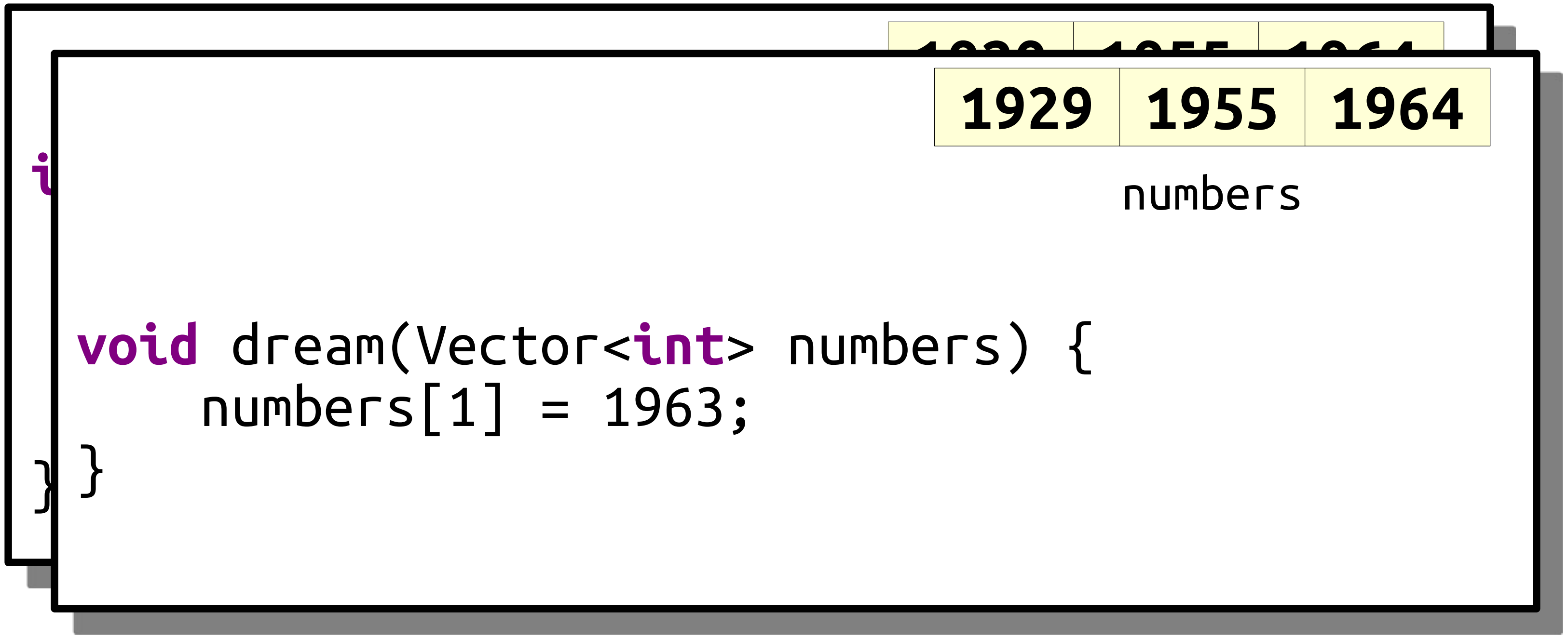
```
int main() {
    Vector<int> values = { 1929, 1955, 1964 };
    dream(values);
    cout << values << endl;
    return 0;
}
```

How it Works



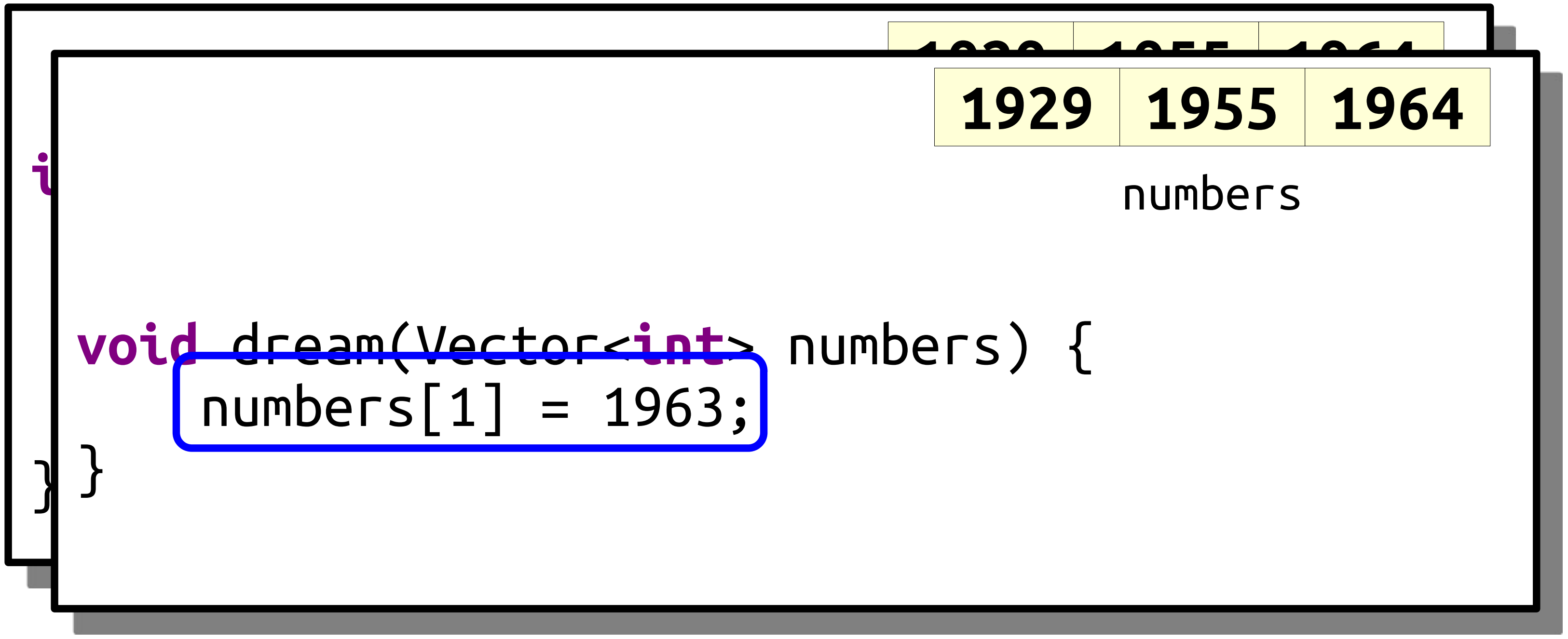
```
int main() {
    Vector<int> values = { 1929, 1955, 1964 };
    dream(values);
    cout << values << endl;
    return 0;
}
```

How it Works



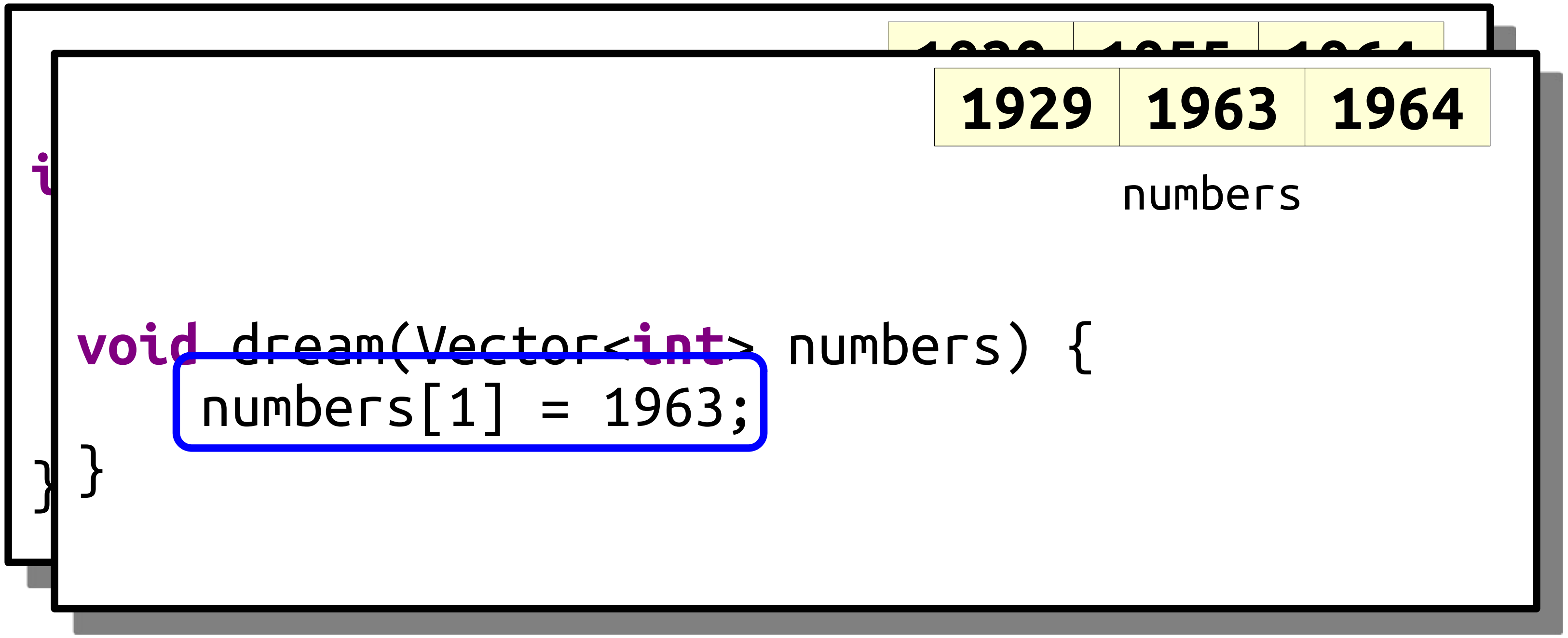
```
void dream(Vector<int> numbers) {
    numbers[1] = 1963;
}
```

How it Works



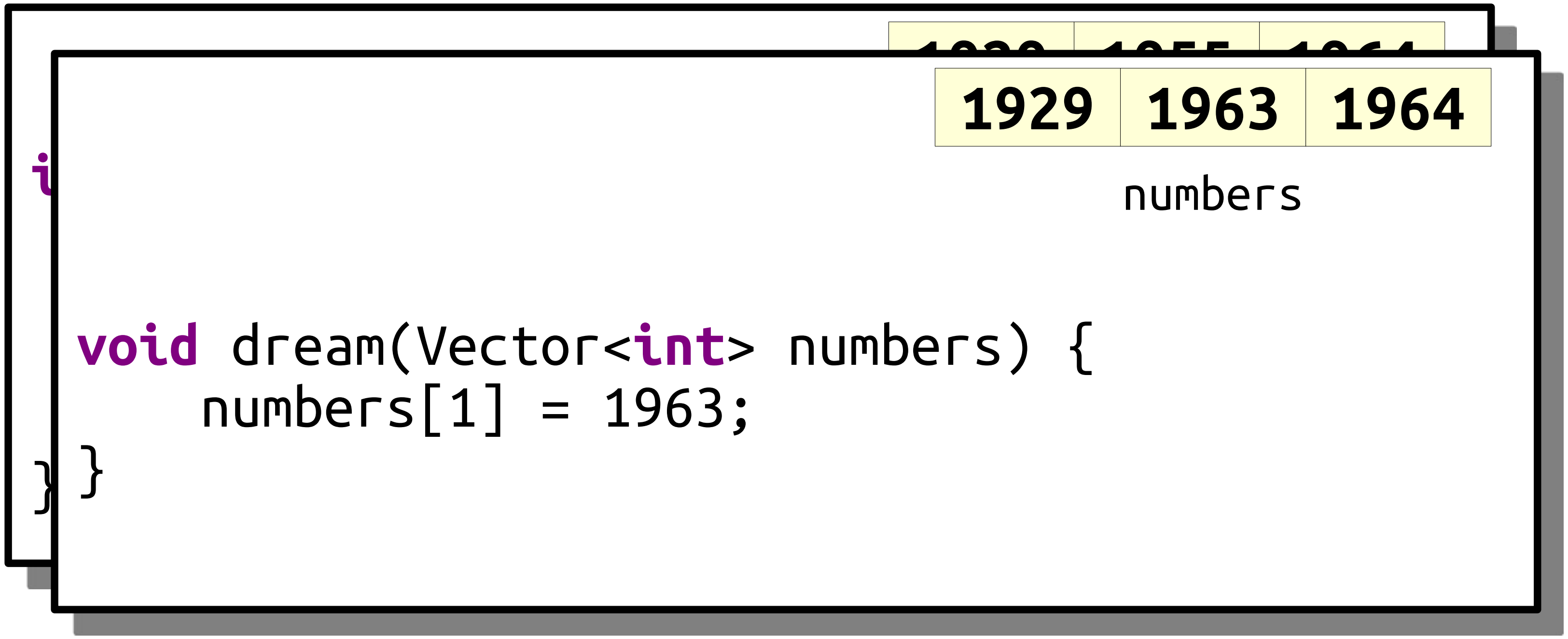
```
void dream(Vector<int> numbers) {
    numbers[1] = 1963;
}
```

How it Works



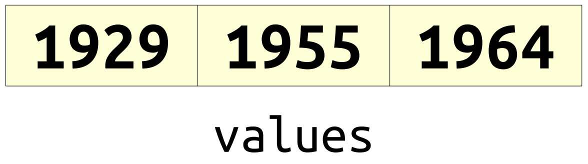
```
void dream(Vector<int> numbers) {
    numbers[1] = 1963;
}
```

How it Works



```
void dream(Vector<int> numbers) {
    numbers[1] = 1963;
}
```

How it Works



```
int main() {
    Vector<int> values = { 1929, 1955, 1964 };
    dream(values);
    cout << values << endl;
    return 0;
}
```

How it Works Now

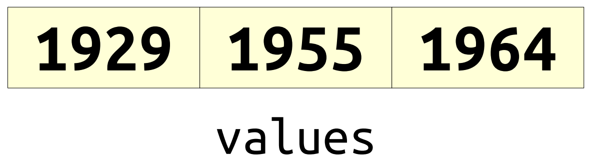
How it Works Now

```
int main() {
    Vector<int> values = { 1929, 1955, 1964 };
    dream(values);
    cout << values << endl;
    return 0;
}
```

How it Works Now

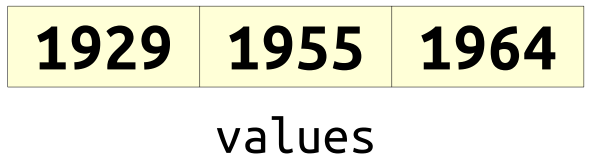
```
int main() {
    Vector<int> values = { 1929, 1955, 1964 };
    dream(values);
    cout << values << endl;
    return 0;
}
```

How it Works Now



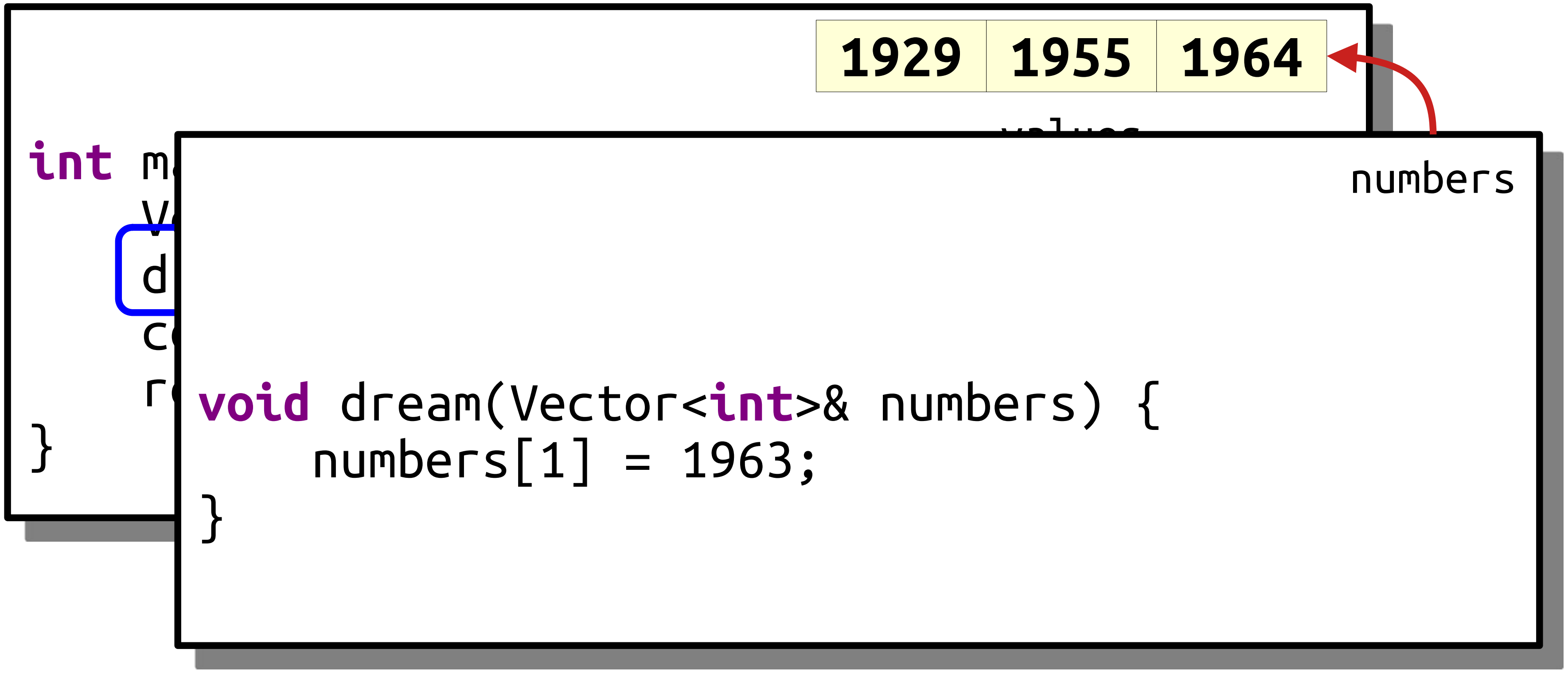
```
int main() {
    Vector<int> values = { 1929, 1955, 1964 };
    dream(values);
    cout << values << endl;
    return 0;
}
```

How it Works Now



```
int main() {
    Vector<int> values = { 1929, 1955, 1964 };
    dream(values);
    cout << values << endl;
    return 0;
}
```

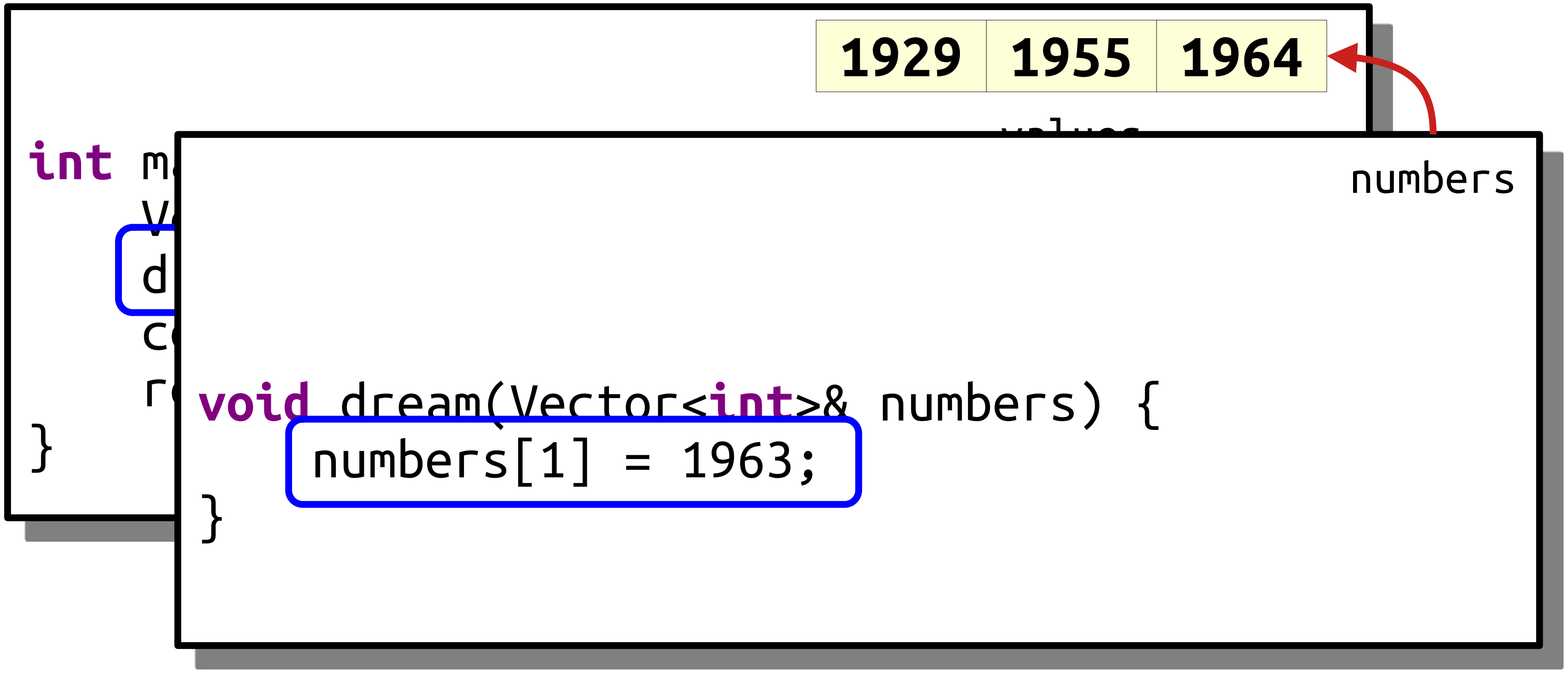
How it Works Now



```
int m
    V
    d
    c
    r
}
```

```
void dream(Vector<int>& numbers) {
    numbers[1] = 1963;
}
```

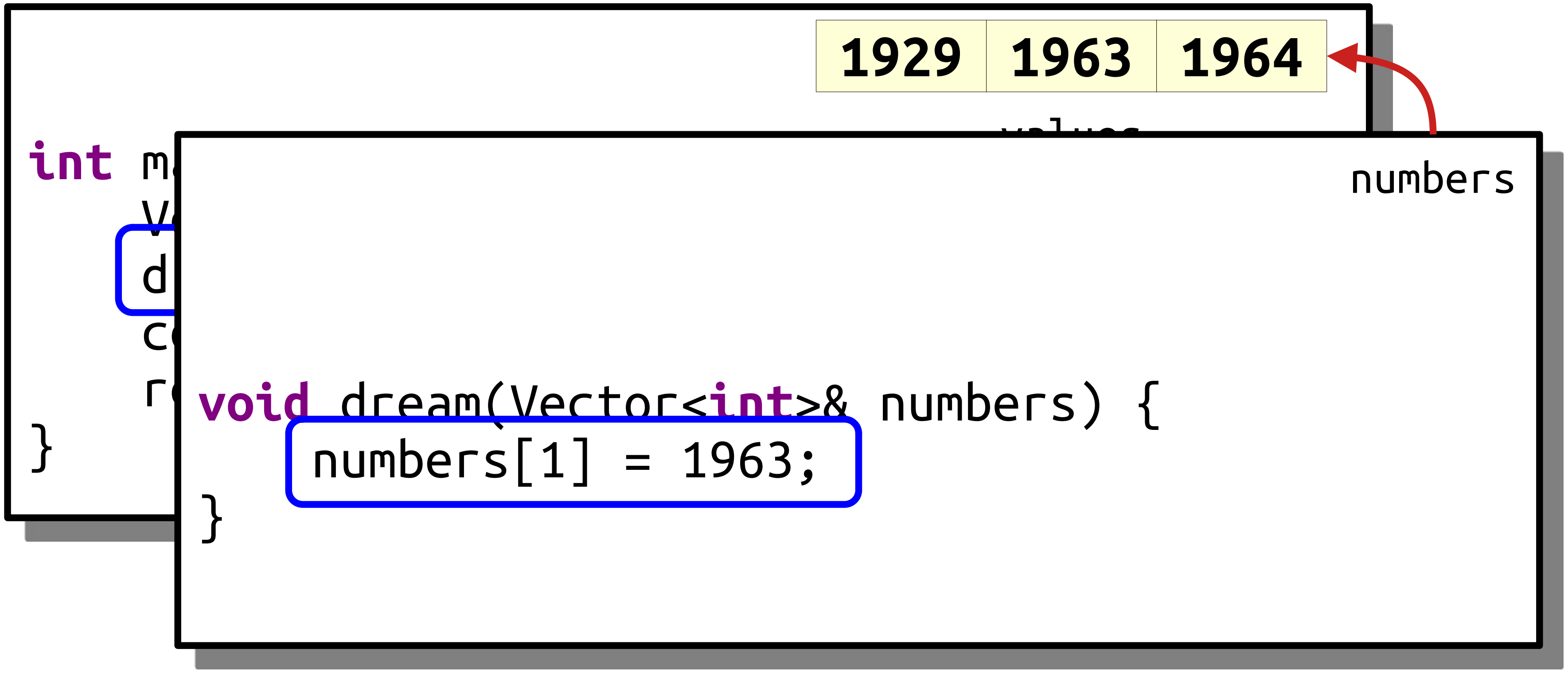
How it Works Now



```
int m
    V
    d
    c
    r
}
```

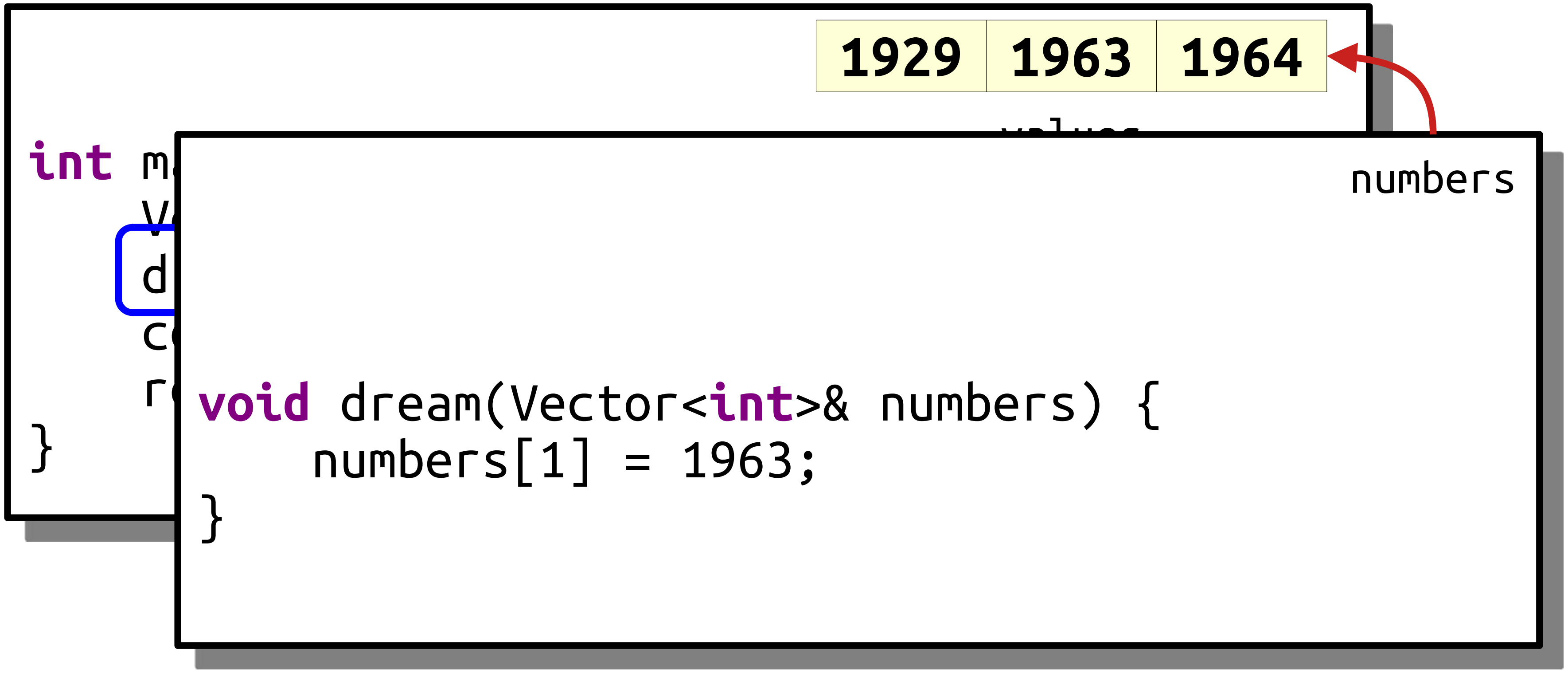
```
void dream(Vector<int>& numbers) {
    numbers[1] = 1963;
}
```

How it Works Now



```
void dream(Vector<int>& numbers) {
    numbers[1] = 1963;
}
```

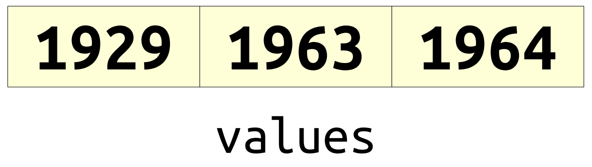
How it Works Now



```
int m
    V
    d
    c
    r
}
```

```
void dream(Vector<int>& numbers) {
    numbers[1] = 1963;
}
```

How it Works Now



```
int main() {
    Vector<int> values = { 1929, 1955, 1964 };
    dream(values);
    cout << values << endl;
    return 0;
}
```

“I’ll live forever! Once the technology is available, I’ll just upload my mind into the cloud.”

How people think it works:

```
void uploadToCloud(Mind& consciousness);
```

How it actually works:

```
void uploadToCloud(Mind consciousness);
```

Credit: Philip Heltweg

Time-Out for Announcements!

Sections

- Discussion sections start this week!
  - Didn’t sign up by Sunday at 5PM? The signup link will reopen on Tuesday at 5PM, and you can choose any open section time.
  - If your section time doesn’t work for you, you can switch into any section with available space starting Tuesday at 5PM. Visit cs198.stanford.edu to do this.
  - Still doesn’t work for you? Ping Neel!
- You’ll get your section assignment this Tuesday at 5:00PM.
- Each week we’ll release a set of section problems on the course website. ***These are not graded***, but we recommend you read over them before your section.

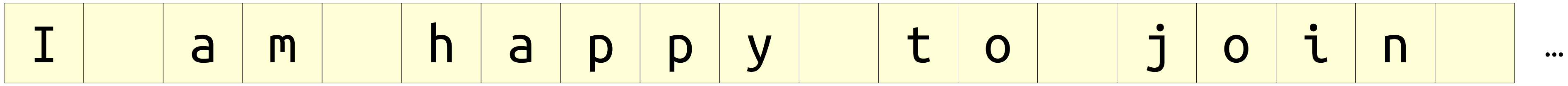
YEAH Hours

- We’ll be holding special sessions called **Y**our **E**arly **A**ssignment **H**elp Hours (YEAH Hours) to give overviews of each of the assignments.
- The first one is today, ***3PM – 4PM*** in ***200-034***.
- These are purely optional, but recommended if you have the bandwidth.

```
return;
```

A Question of Speed

- When working with strings or containers, pass-by-value is slower than pass-by-reference because of the cost of copying data.



- ***General principle:*** When passing a string or container into a function, use pass-by-reference unless you actually want a copy.

Do You Trust Me?

- Suppose you’ve written the next Great American Novel and the single, sole copy is stored in the variable

```
string myMasterpiece;
```

- You see a function with this signature:

```
void totallyNotSketchy(string& text);
```

- Would you make this call?

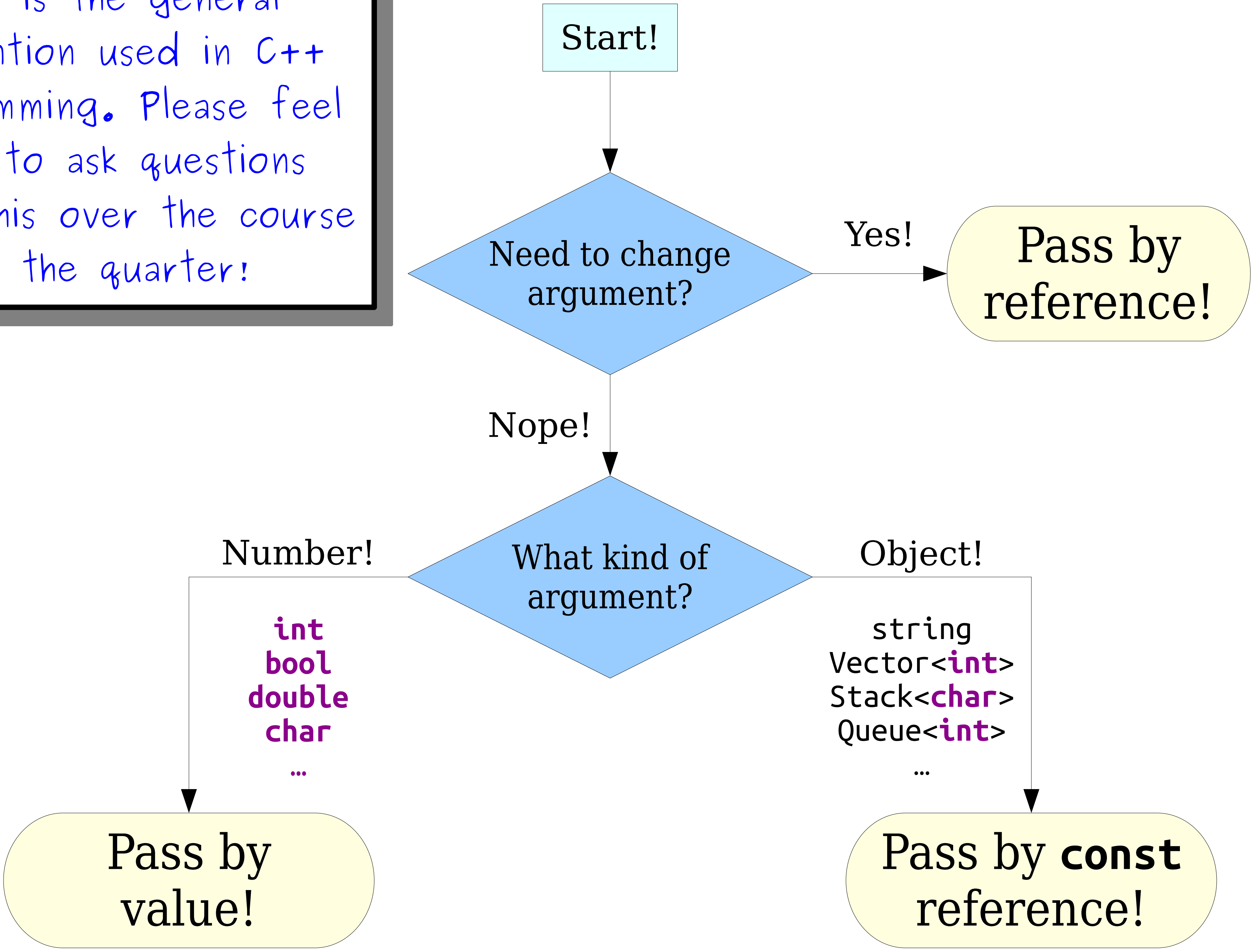
```
totallyNotSketchy(myMasterpiece);
```

Pass-by-const-Reference

- If you want to look at, but not modify, a function parameter, pass it by ***const reference***:
  - The "by reference" part avoids a copy.
  - The "**const**" (constant) part means that the function can't change that argument.
- For example:

```
void proofreadLongEssay(const string& essay) {
   /* can read, but not change, the essay.  */
}
```

This is the general convention used in C++ programming. Please feel free to ask questions about this over the course of the quarter!



Recursion on `Vectors`

Finding the Largest Number

Finding the Largest Number

- Our goal is to write a function

  ```
  int maxOf(const Vector<int>& numbers);
  ```

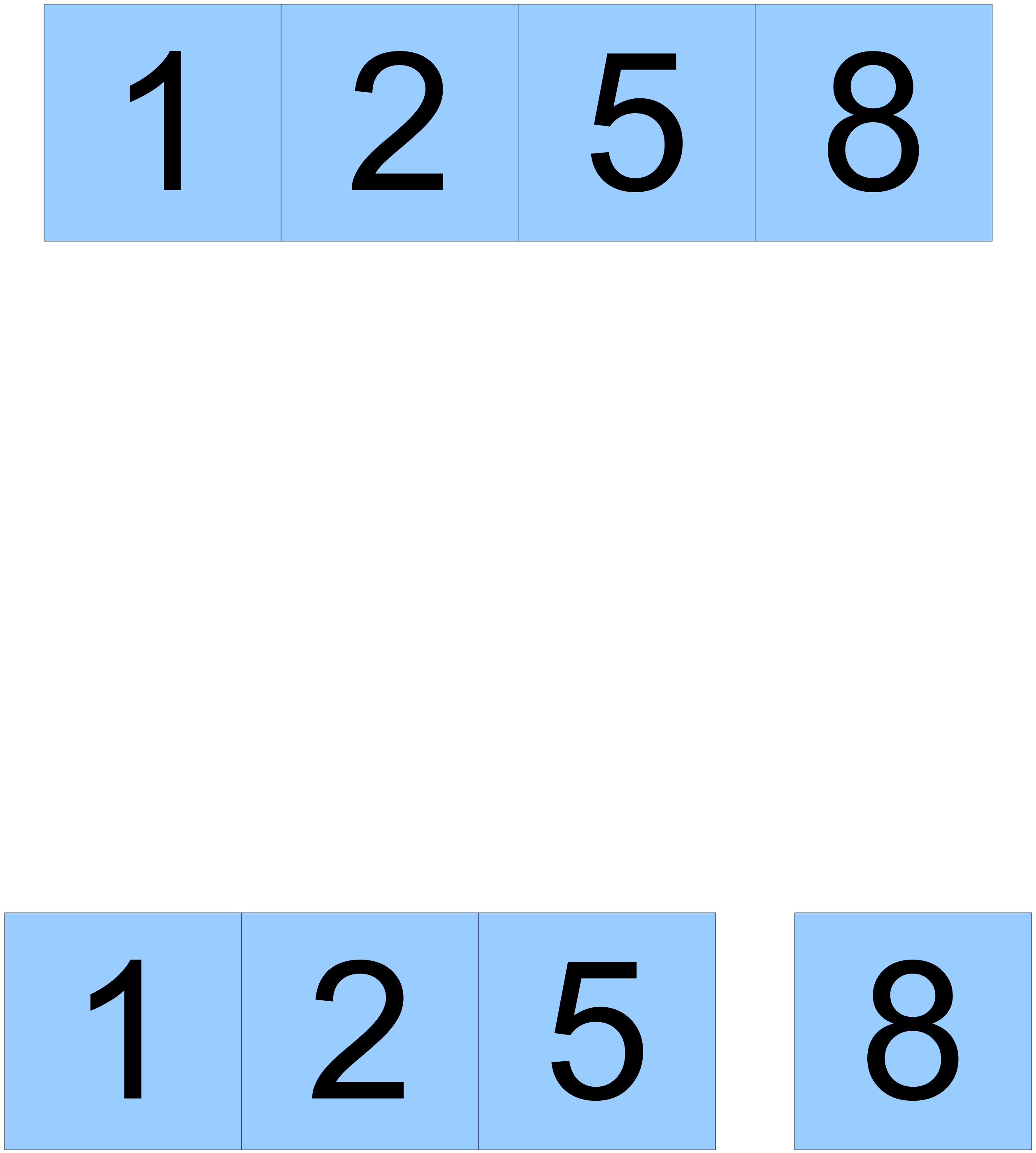
  that takes as input a `Vector<int>`, then returns the largest number in the `Vector`.
- We're going to assume the `Vector` has at least one element in it; otherwise, it's not possible to return the largest value!
- Let's see how to do this.

Thinking Recursively

```
if (The problem is very simple) {
    Directly solve the problem.
    Return the solution.
} else {
    Split the problem into one or more
    smaller problems with the same
    structure as the original.
    Solve each of those smaller problems.
    Combine the results to get the overall
    solution.
    Return the overall solution.
}
```

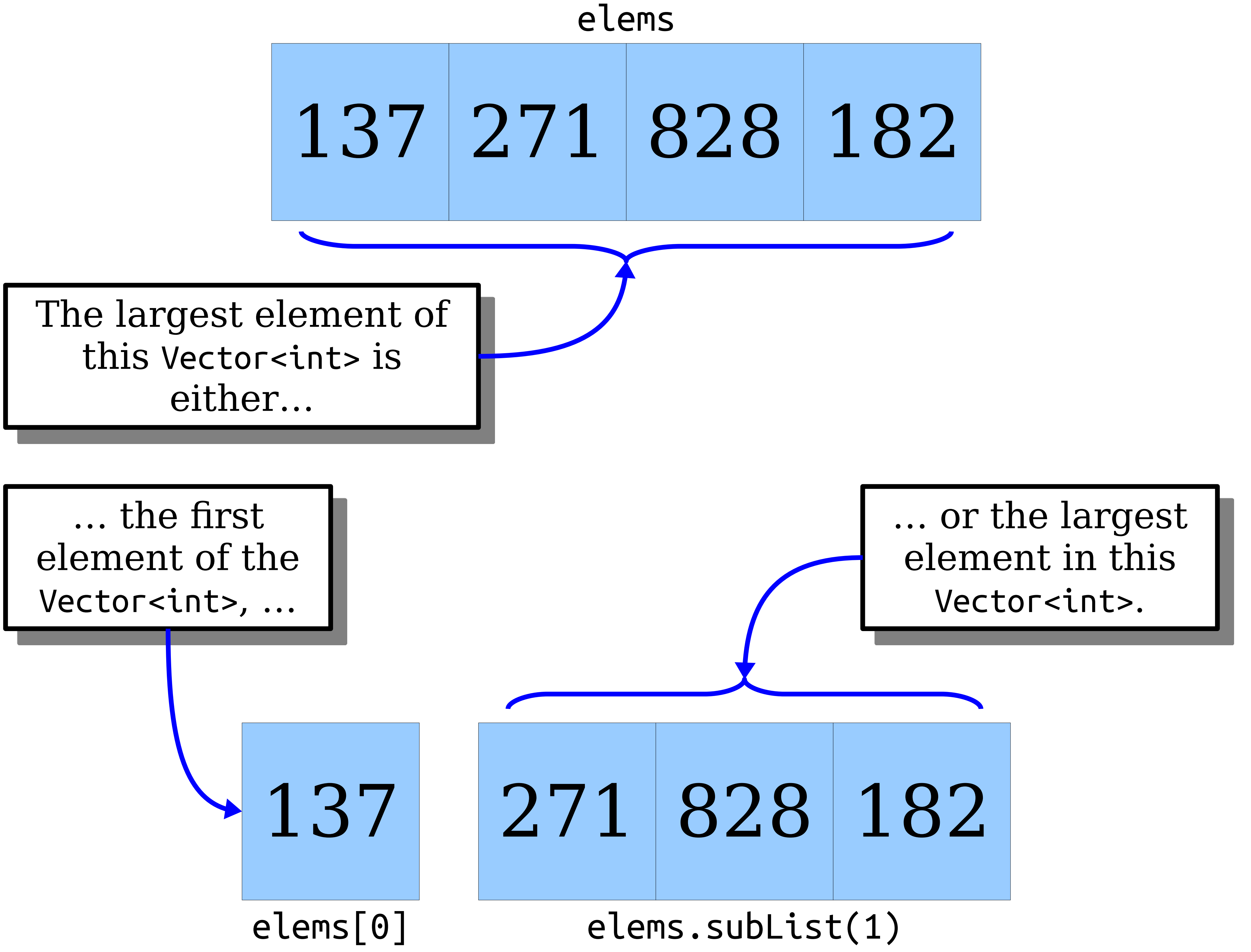
These simple cases are called *base cases*.

These are the *recursive cases*.


1
2
5
8
1
2
5
8

I B E X

I B E X


elems
137
271
828
182
The largest element of this Vector<int> is either...
... the first element of the Vector<int>, ...
... or the largest element in this Vector<int>.
137
271
828
182
elems[0]
elems.subList(1)

Tracing the Recursion

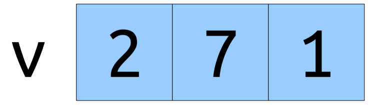
```
int main() {
   Vector<int> v = { 2, 7, 1 };
   cout << maxOf(v) << endl;
   return 0;
}
```

Tracing the Recursion

```
int main() {
    Vector<int> v = { 2, 7, 1 };
    cout << maxOf(v) << endl;
    return 0;
}
```

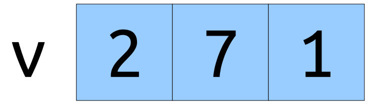
Tracing the Recursion

```
int main() {
    Vector<int> v = { 2, 7, 1 };
    cout << maxOf(v) << endl;
    return 0;
}
```



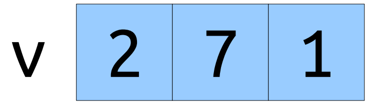
Tracing the Recursion

```
int main() {
    Vector<int> v = { 2, 7, 1 };
    cout << maxOf(v) << endl;
    return 0;
}
```



Tracing the Recursion

```
int main() {
    Vector<int> v = { 2, 7, 1 };
    cout << maxOf(v) << endl;
    return 0;
}
```



Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

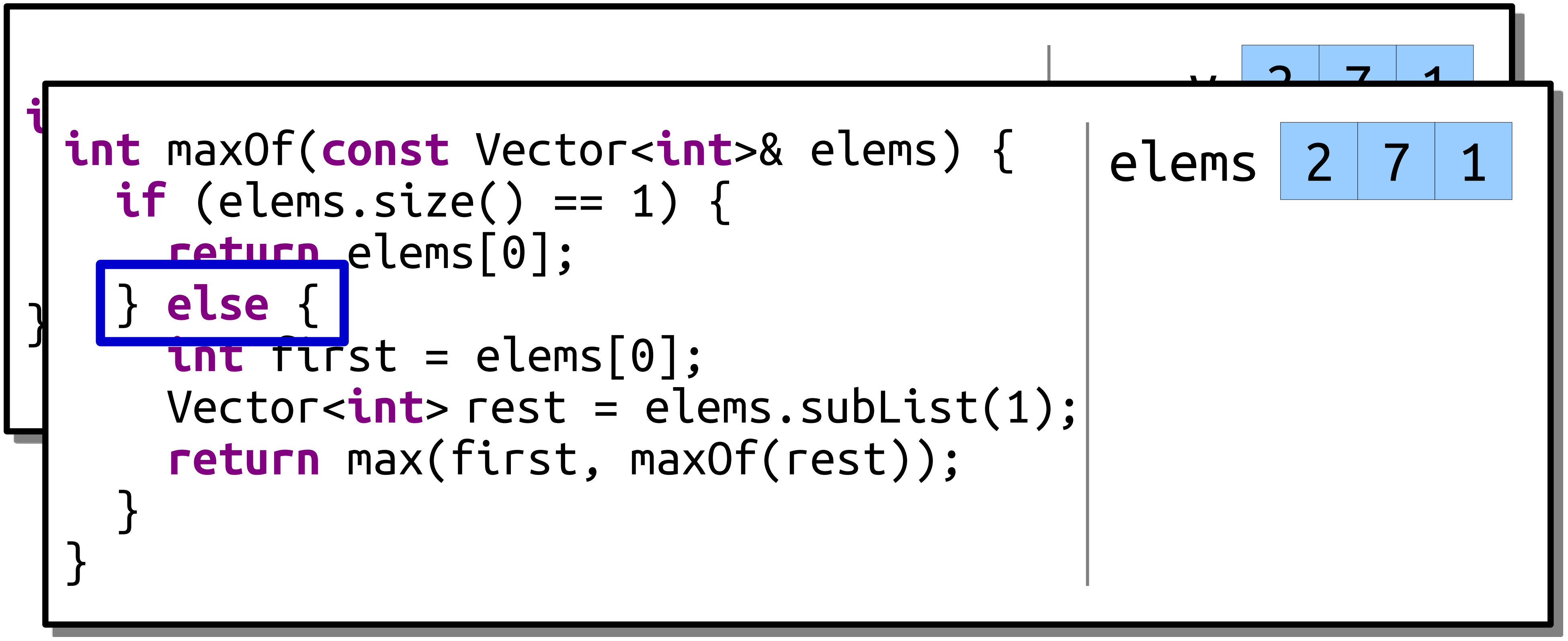
elems	2	7	1

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

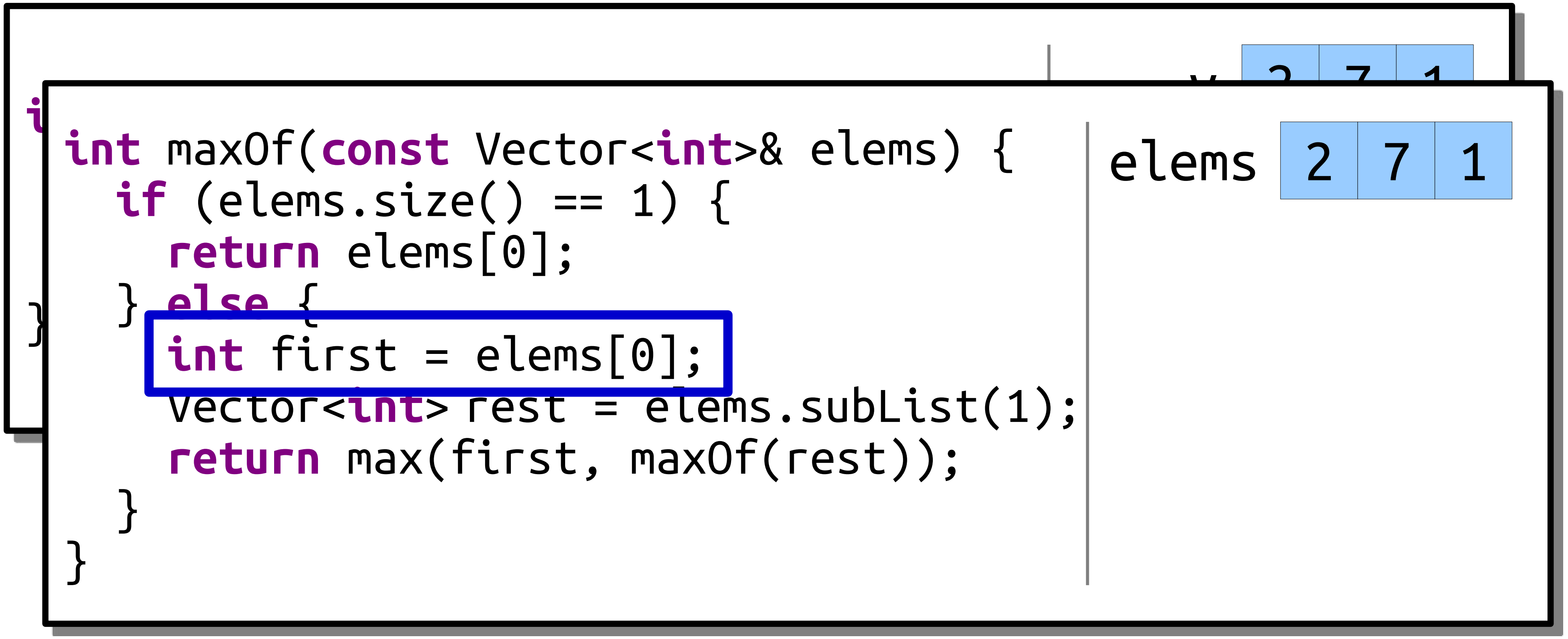
elems	2	7	1

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

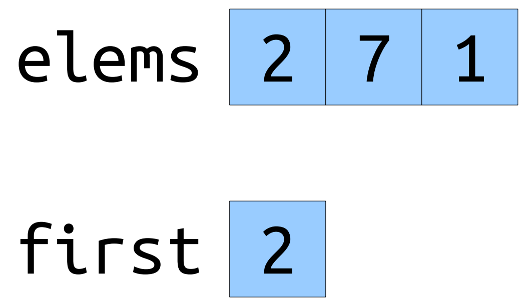
Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

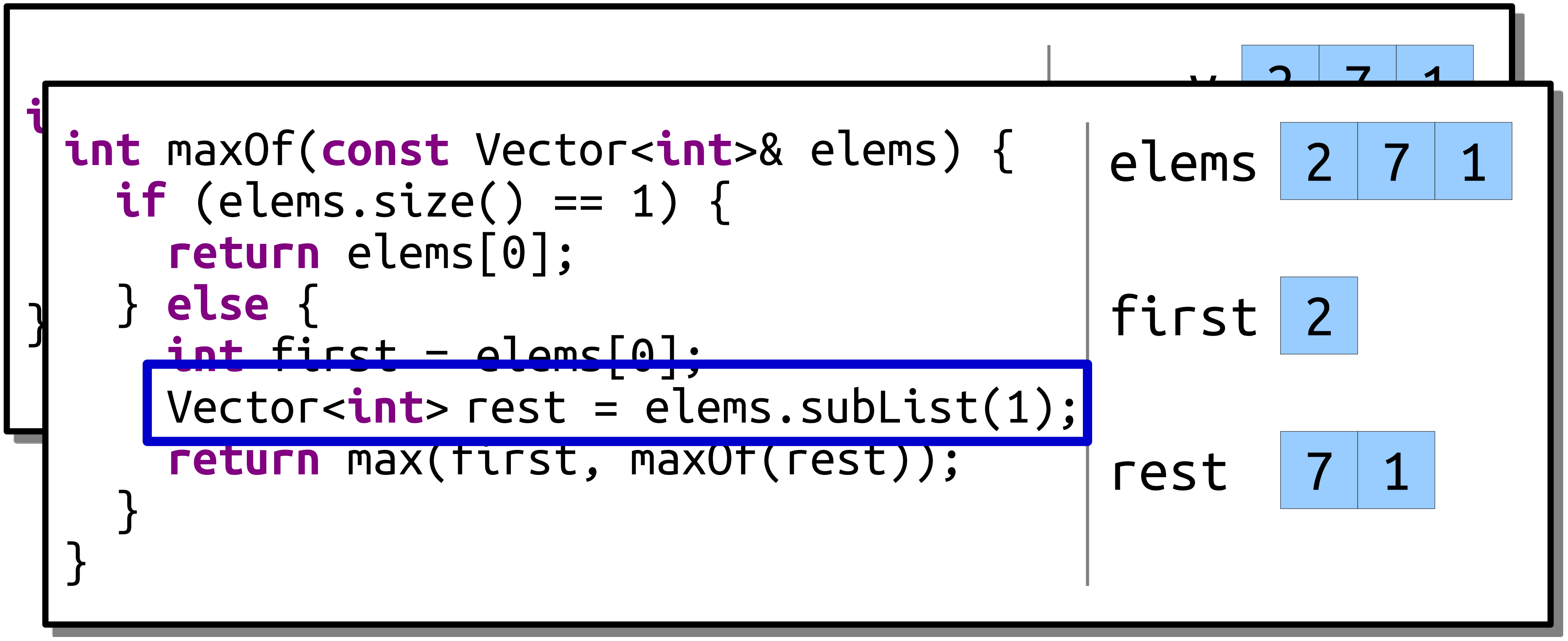


Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

elems	2	7	1
first	2		

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

elems	2	7	1
first	2		
rest	7	1	

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

elems	2	7	1
first	2		
rest	7	1	

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

2

elems	2	7	1
first	2		
rest	7	1	

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

2

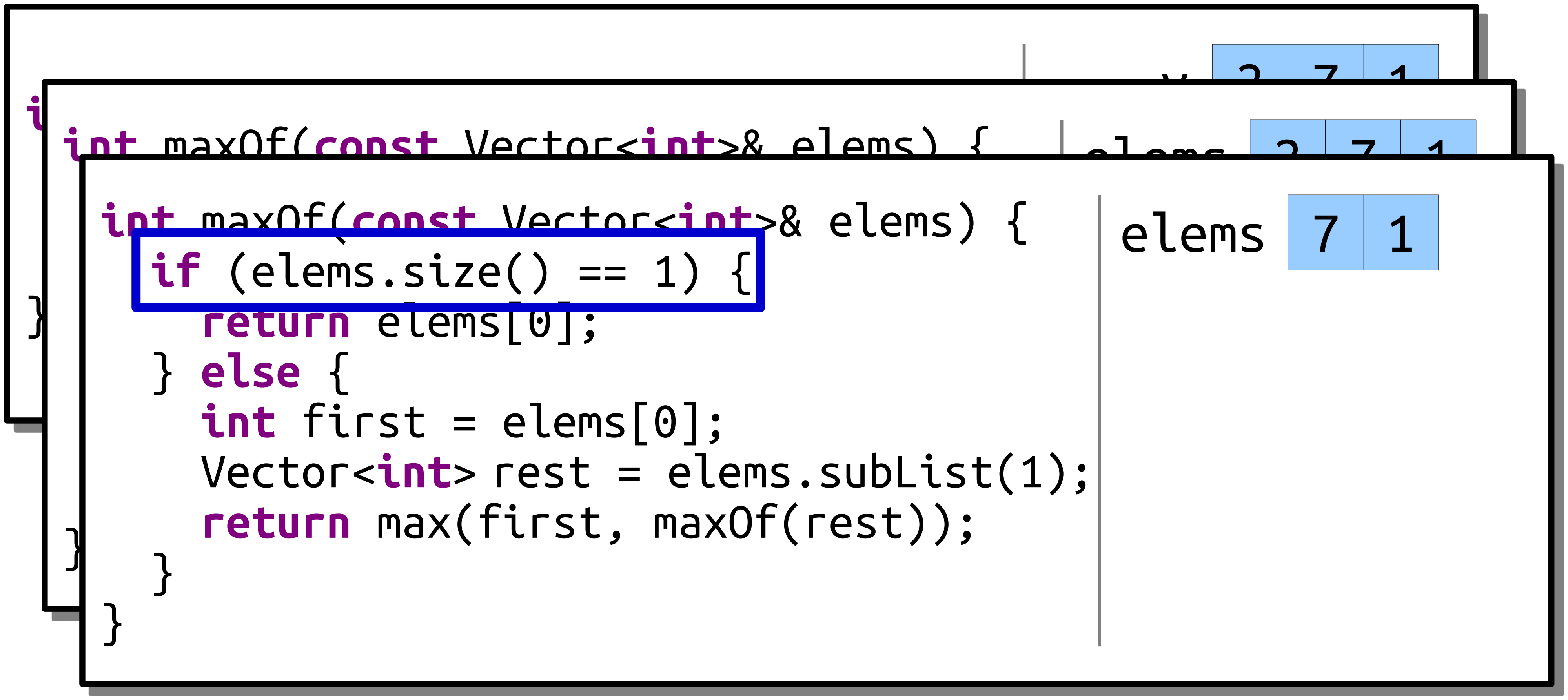
elems	2	7	1
first	2		
rest	7	1	

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

elems	7	1

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
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}
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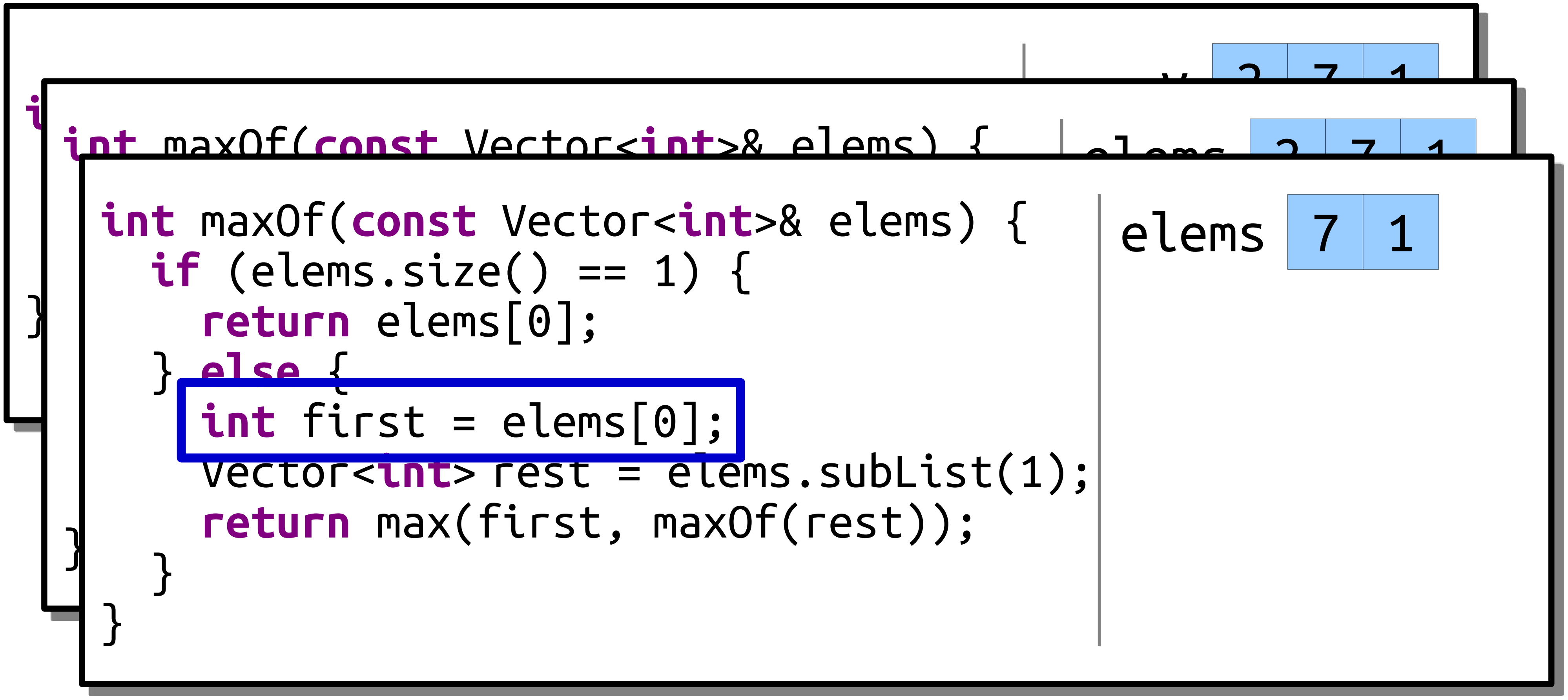
Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

elems	7	1

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

elems	7	1

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

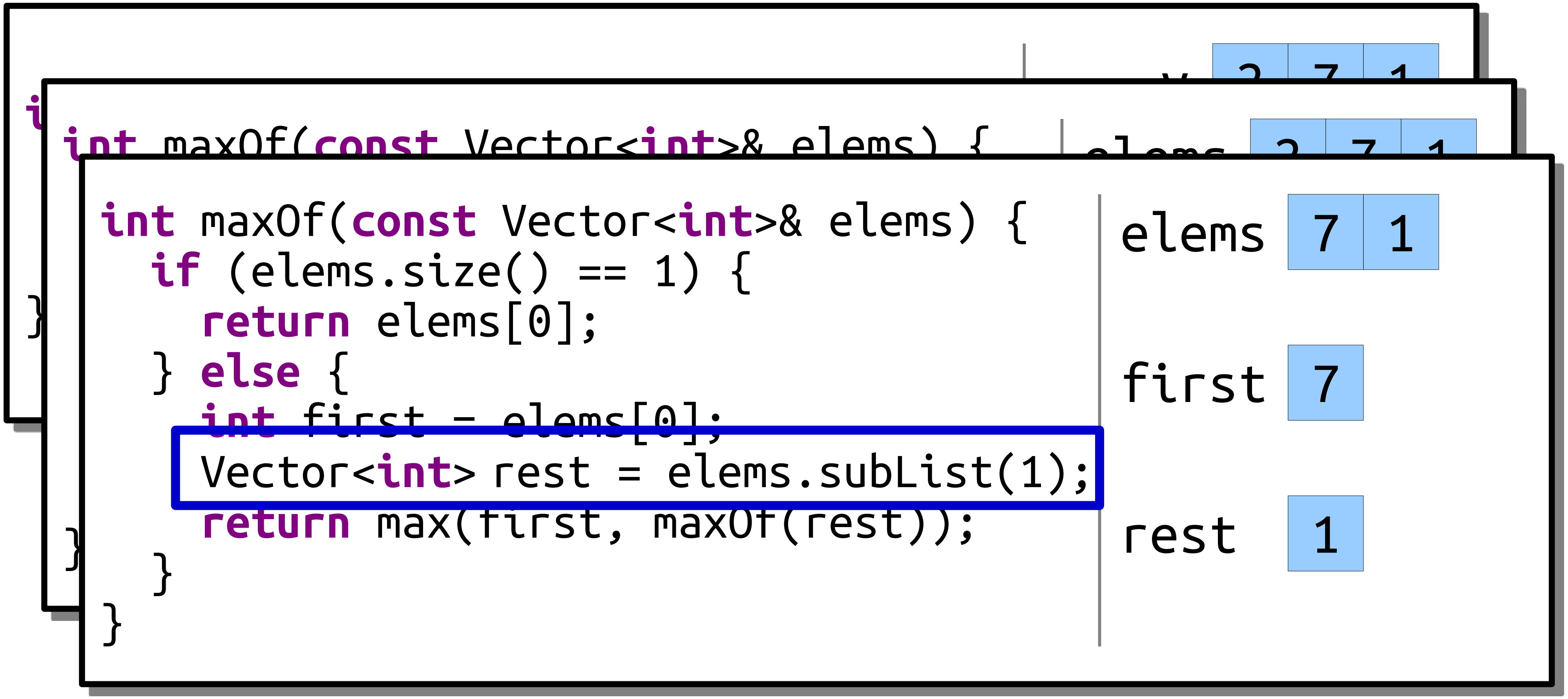
elems	7	1
first	7	

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

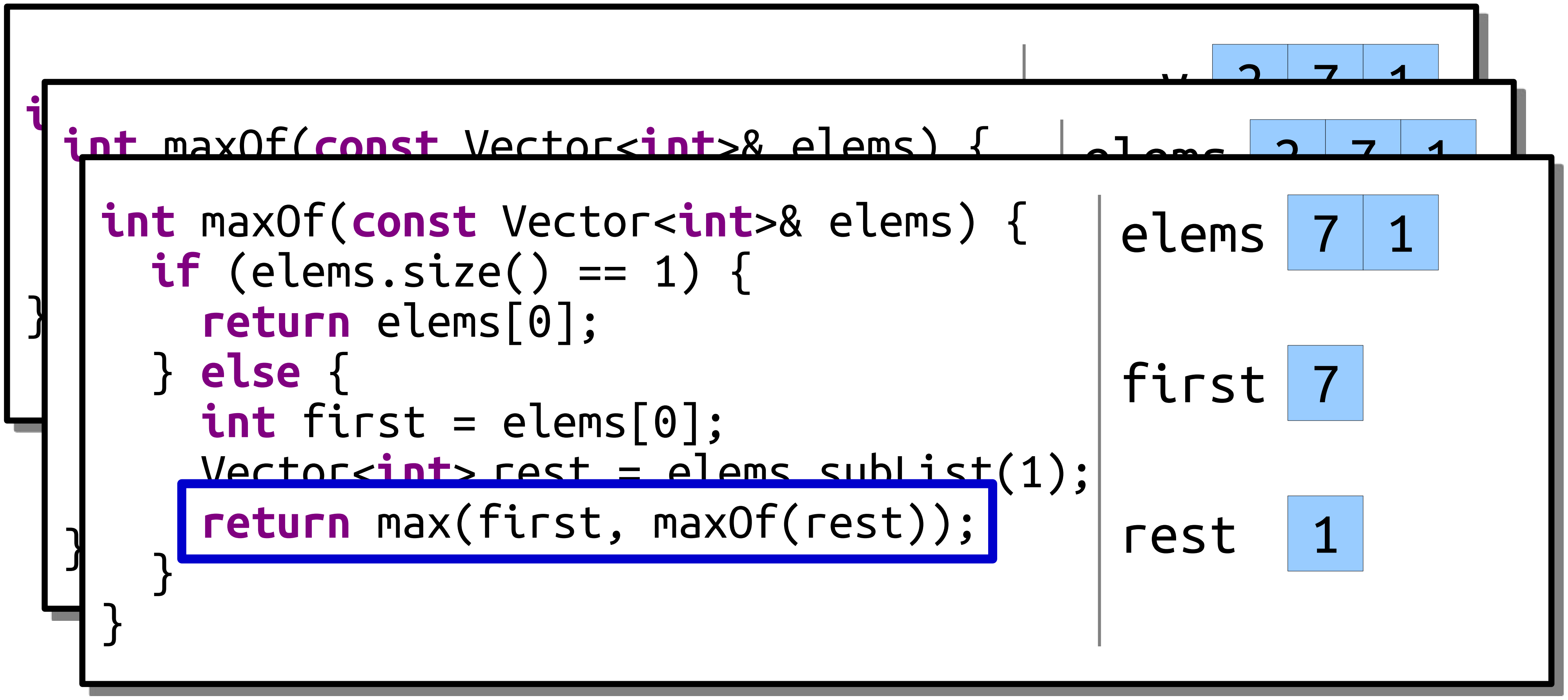
elems	7	1
first	7	

Tracing the Recursion



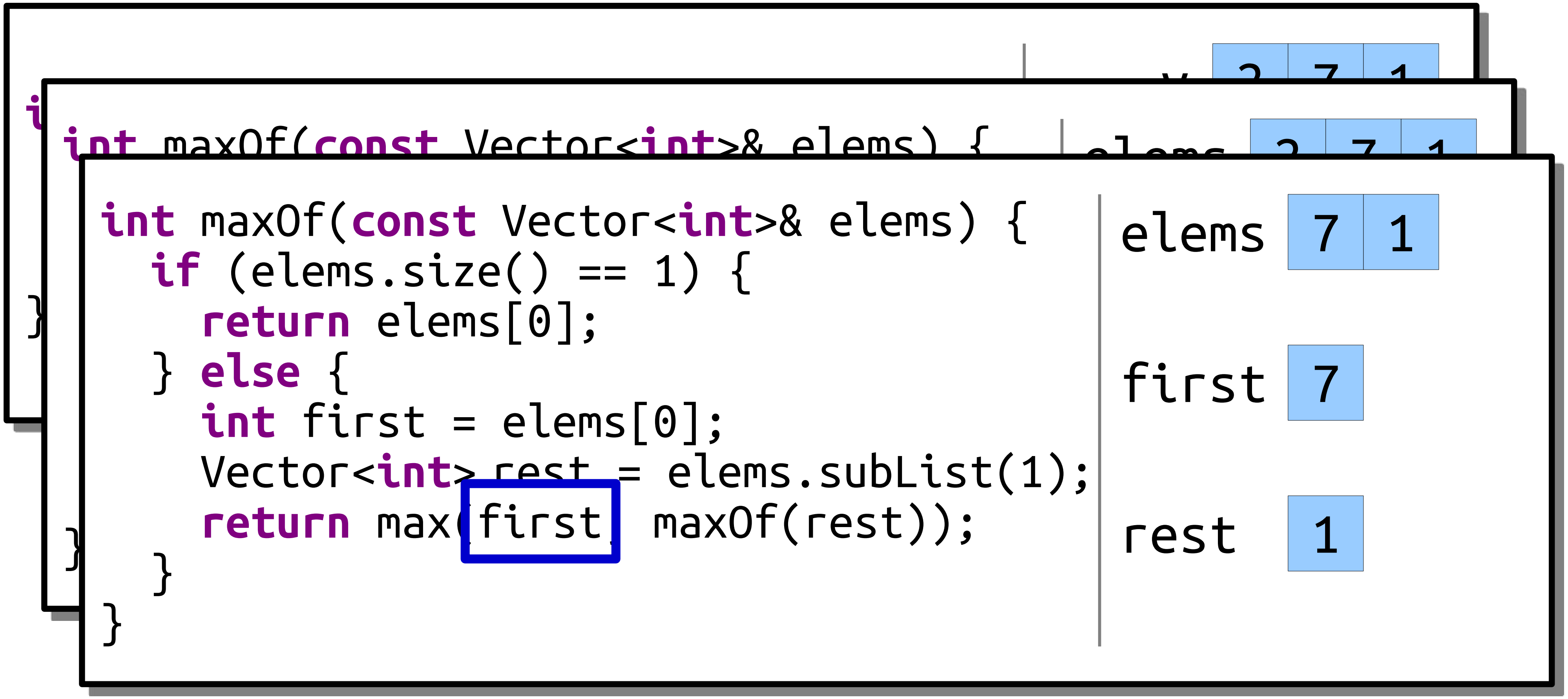
```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

Tracing the Recursion



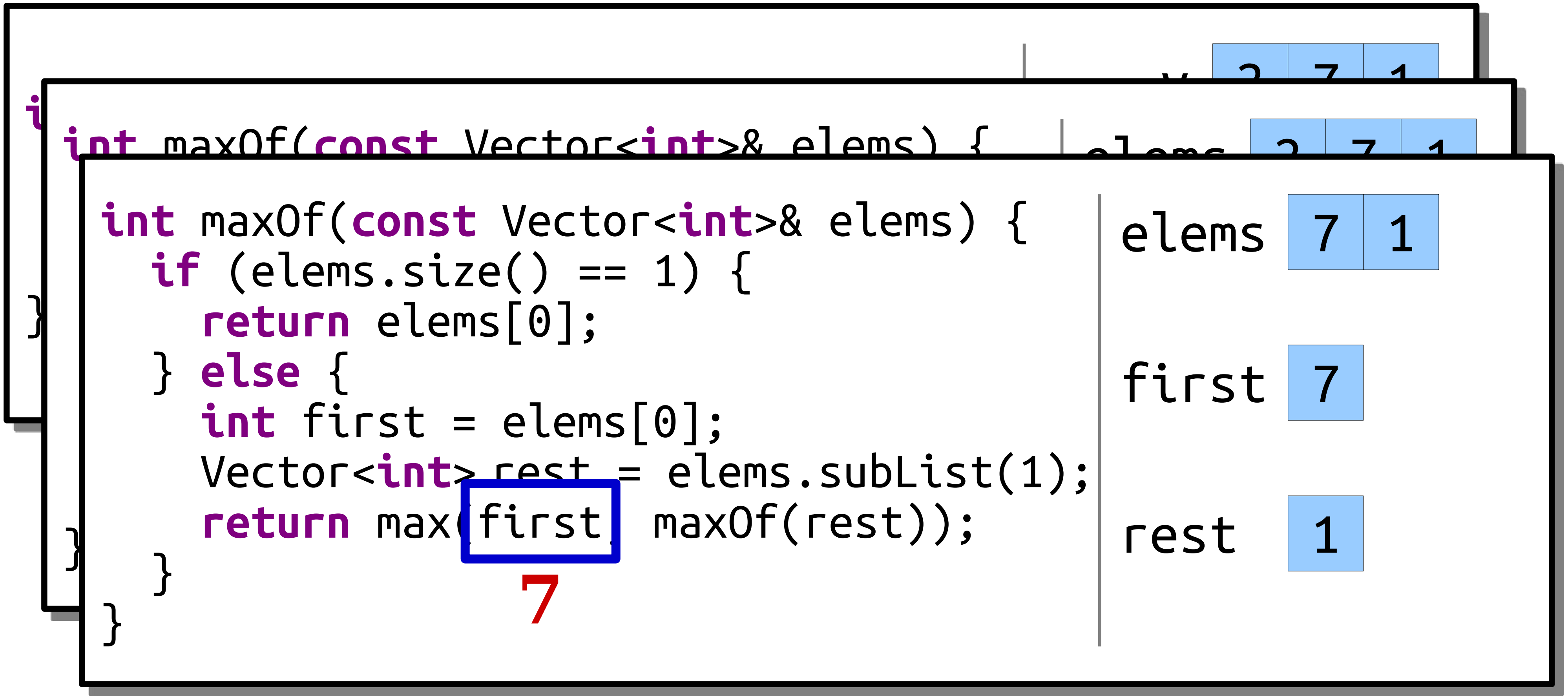
```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.sublist(1);
    return max(first, maxOf(rest));
  }
}
```

7

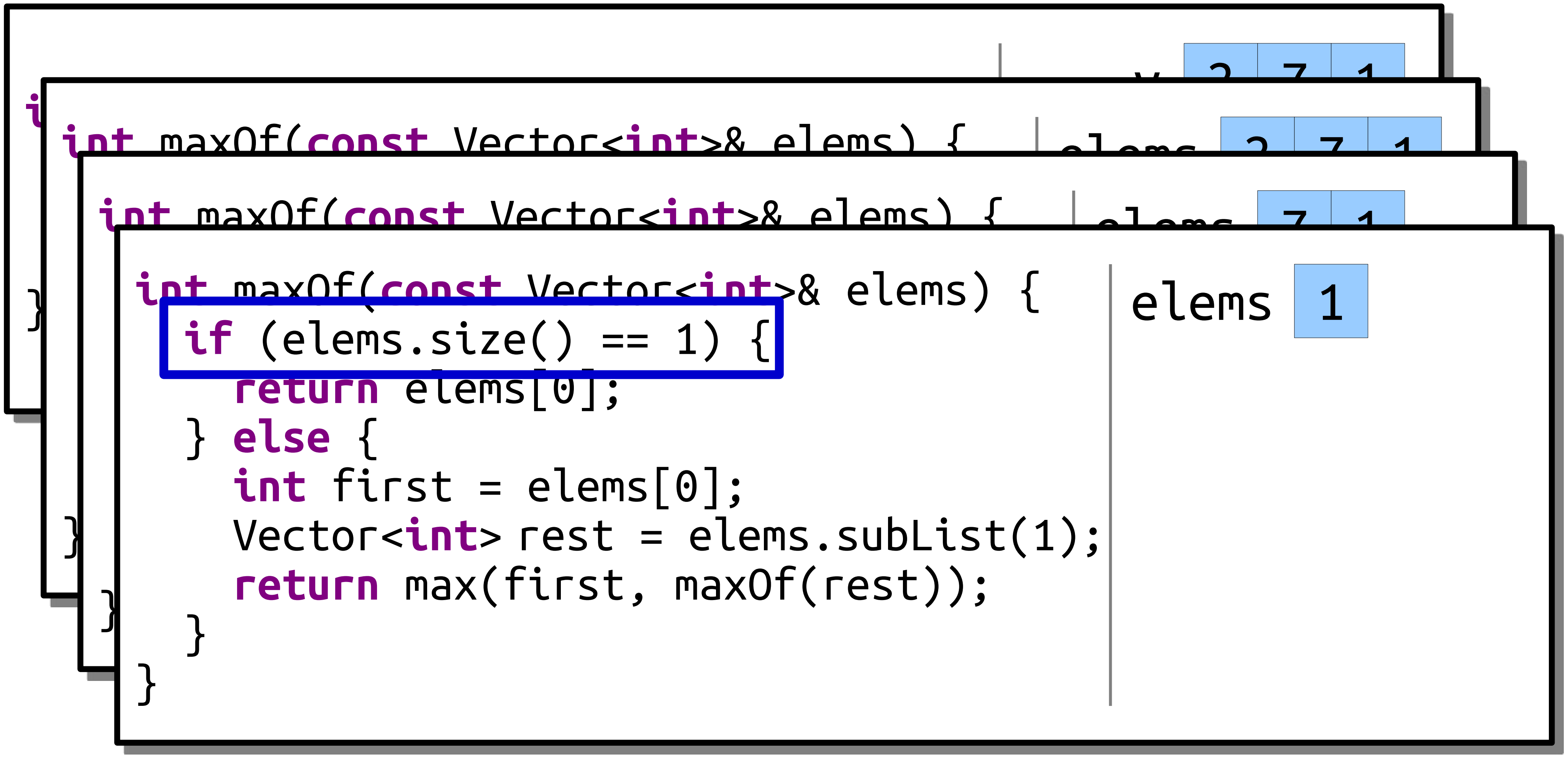
elems	7	1
first	7	
rest	1	

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

elems | 1 |

Tracing the Recursion



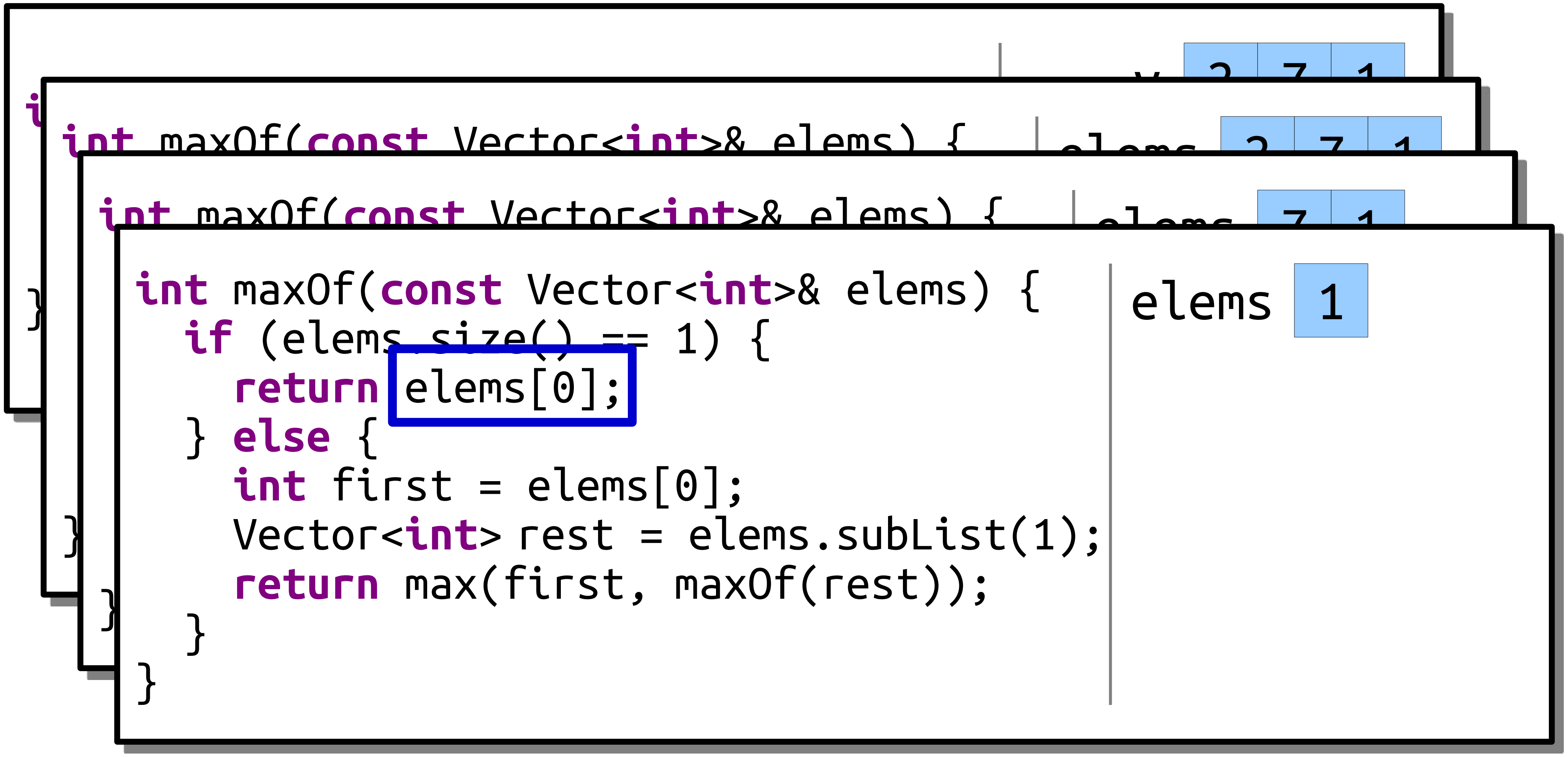
```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

elems	1

Tracing the Recursion



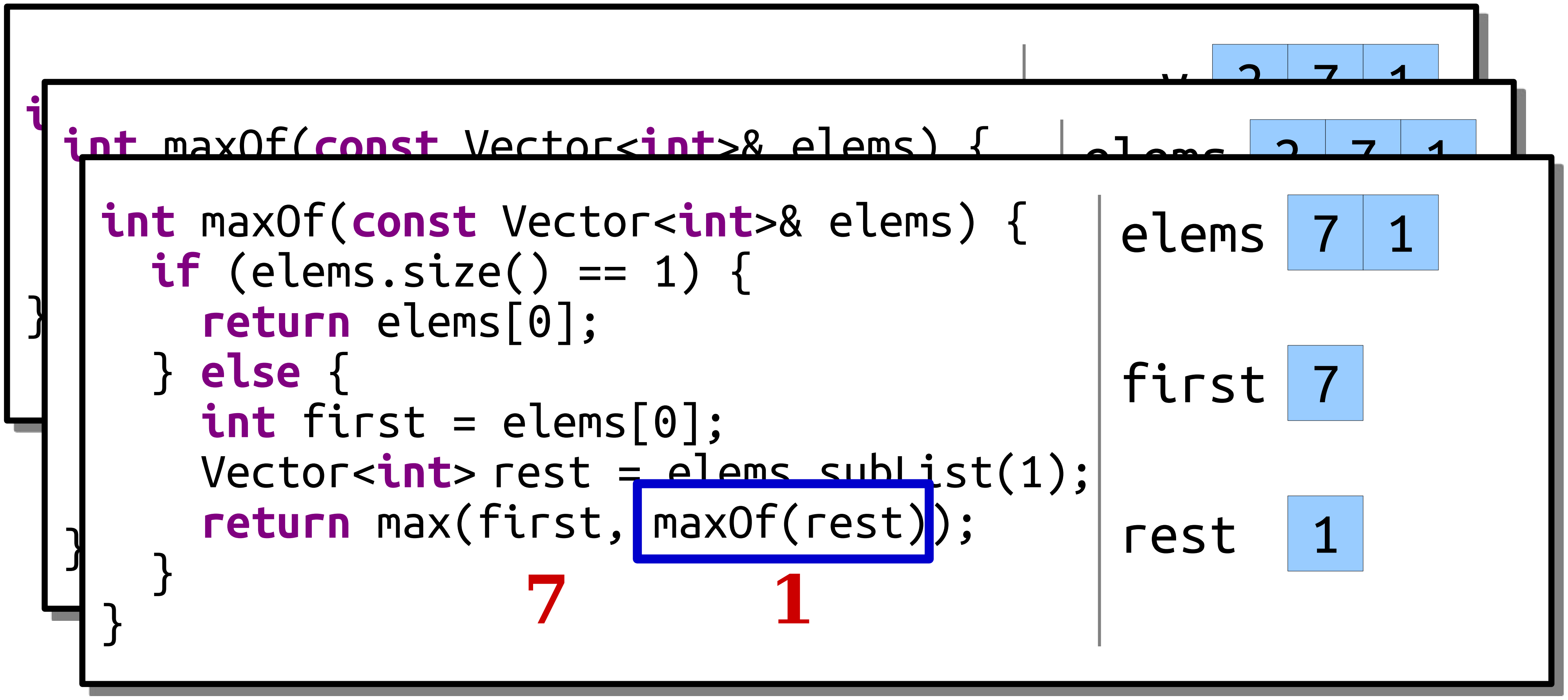
```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];  1
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

elems | 1

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.sublist(1);
    return max(first, maxOf(rest));
  }
}
```

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.sublist(1);
    return max(first, maxOf(rest));
  }
}
```

$\boxed{\texttt{max(first, maxOf(rest));}}$

7 1

elems	7	1
first	7	
rest	1	

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.sublist(1);
    return max(first, maxOf(rest));
  }
}
```

7

elems	7	1
first	7	
rest	1	

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.sublist(1);
    return max(first, maxOf(rest));
  }
}
```

7

elems	7	1
first	7	
rest	1	

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

2 7

elems	2	7	1
first	2		
rest	7	1	

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

$\underbrace{\text{first}}_{2}$, $\underbrace{\text{maxOf(rest)}}_{7}$

elems	2	7	1
first	2		
rest	7	1	

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

7

elems	2	7	1
first	2		
rest	7	1	

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int first = elems[0];
    Vector<int> rest = elems.subList(1);
    return max(first, maxOf(rest));
  }
}
```

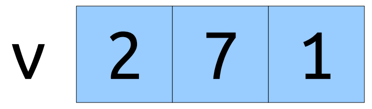
7

elems	2	7	1
first	2		
rest	7	1	

Tracing the Recursion

```
int main() {
    Vector<int> v = { 2, 7, 1 };
    cout << maxOf(v) << endl;
    return 0;
}
```

7



Summary from Today

- The `Vector<T>` type in C++ represents a sequence of elements.
- Parameters in C++ are passed by *value* by default. You can change that to use pass by *reference* if you'd like.
- Use pass-by-**`const`**-reference for objects you don't intend to change.
- Each stack frame from a recursive function gets its own copies of all the local variables.

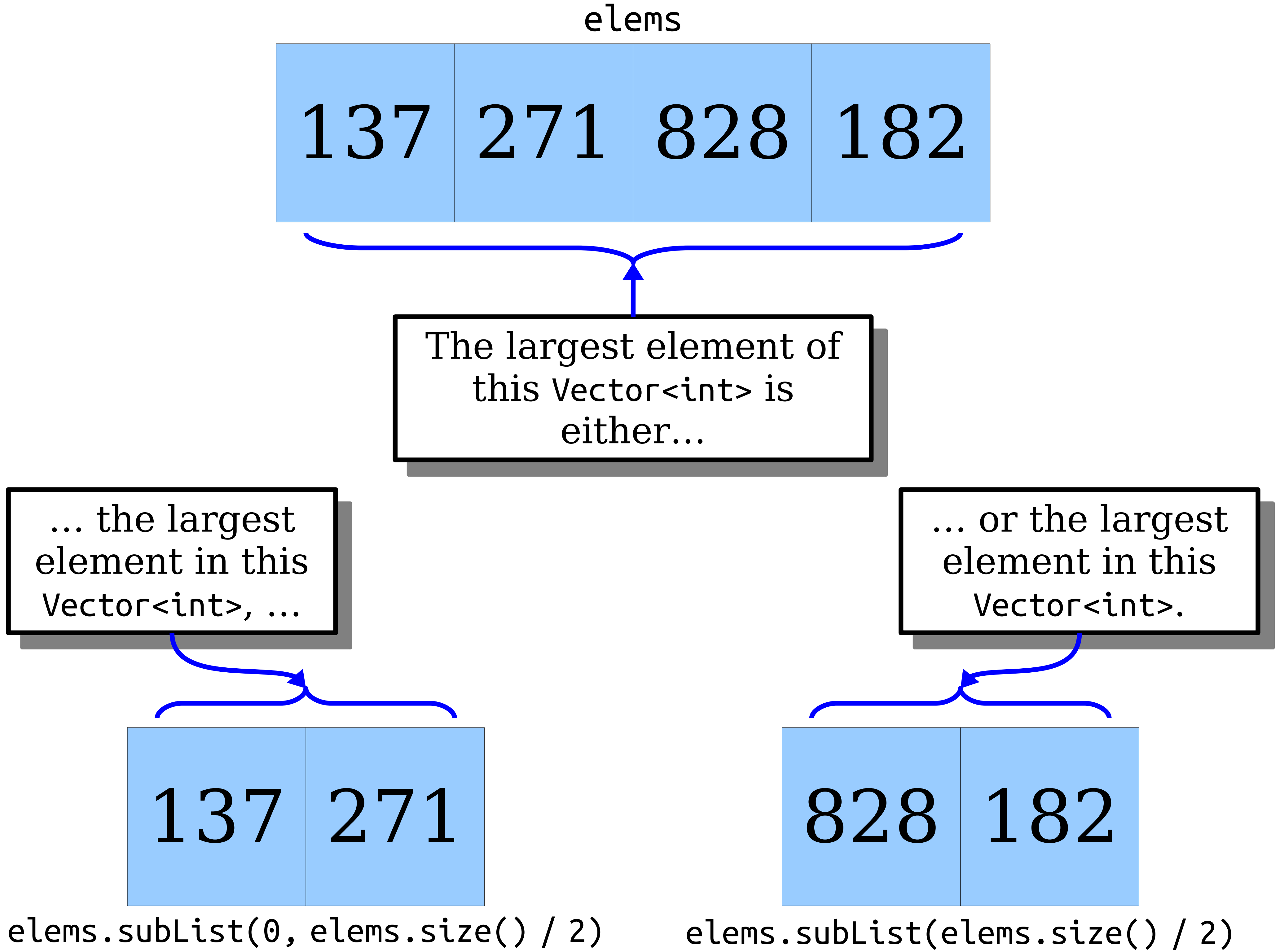
Your Action Items

- ***Read Chapter 5.1 and Chapter 5.2 of the textbook.***
  - It's all about `Vector` and `Grid`! There are some goodies there.
- ***Work on Assignment 1.***
  - If you're following our recommended timetable, aim to have Debugger Warmups and Fire completed tonight, and start working on Only Connect by Wednesday.
- ***Explore the `maxOf` example.***
  - Tinker and play around with this one. See if you can get very comfortable with how it works.

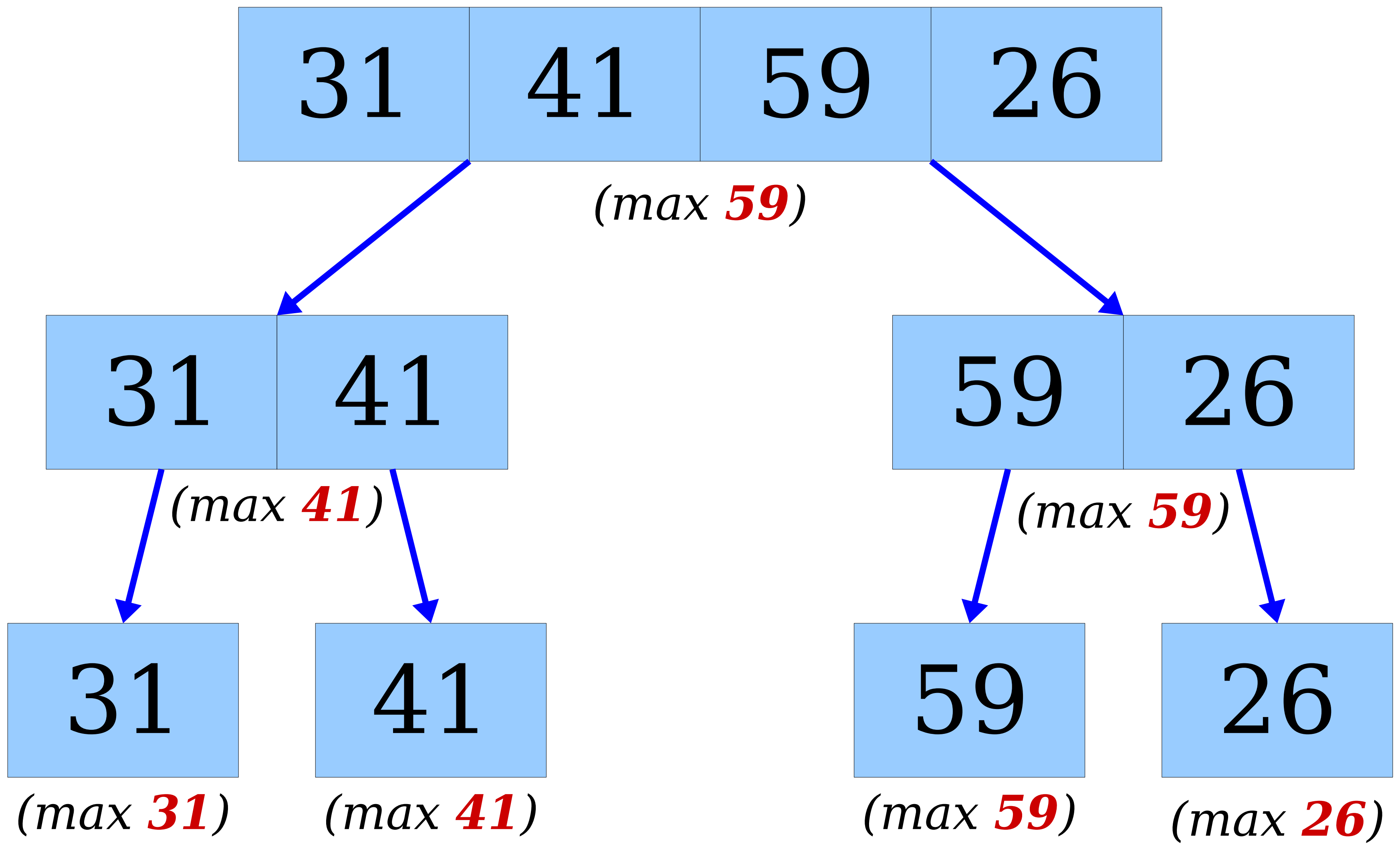
Next Time

- ***Lexicons, Sets, and Maps.***
  - Storing words.
  - Storing items in No Particular Order.
  - Associating items with one another.
- ***Fun With Words***
  - Simple programs + rich data = cool demos.

Appendix: Finding the max, another way.


elems
137
271
828
182
The largest element of this Vector<int> is either…
… the largest element in this Vector<int>, …
… or the largest element in this Vector<int>.
137
271
828
182
elems.subList(0, elems.size() / 2)
elems.subList(elems.size() / 2)

`maxOf` as a Tournament



maxOf as a Tournament

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;

    /* First half of the elements. */
    Vector<int> left  = elems.subList(0, half);

    /* Second half of the elements. */
    Vector<int> right = elems.subList(half);

    /* Biggest value in the overall list is either
     * the largest element in the first half or
     * the largest element in the second half.
     */
    return max(maxOf(left), maxOf(right));
  }
}
```

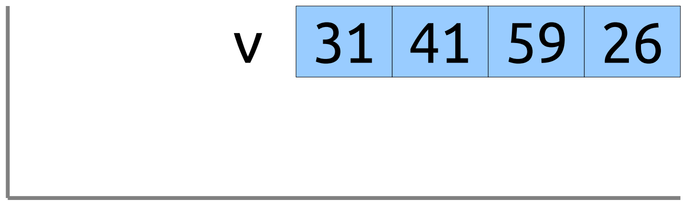
Tracing the Recursion

```
int main() {
   Vector<int> v = { 31, 41, 59, 26 };
   cout << maxOf(v) << endl;
   return 0;
}
```

Tracing the Recursion

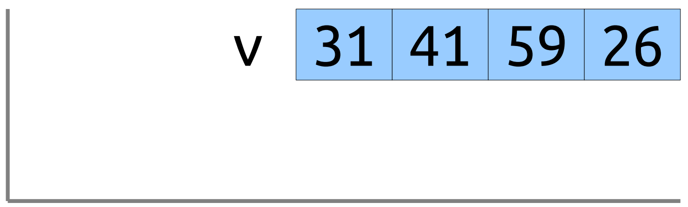
```
int main() {
    Vector<int> v = { 31, 41, 59, 26 };
    cout << maxOf(v) << endl;
    return 0;
}
```

Tracing the Recursion



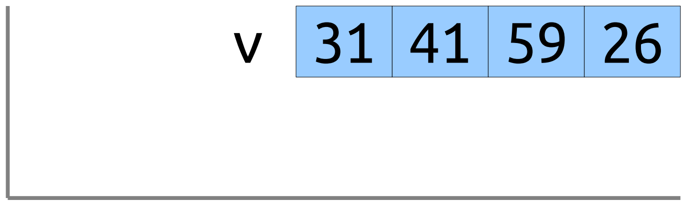
```
int main() {
   Vector<int> v = { 31, 41, 59, 26 };
   cout << maxOf(v) << endl;
   return 0;
}
```

Tracing the Recursion



```
int main() {
    Vector<int> v = { 31, 41, 59, 26 };
    cout << maxOf(v) << endl;
    return 0;
}
```

Tracing the Recursion



```
int main() {
    Vector<int> v = { 31, 41, 59, 26 };
    cout << maxOf(v) << endl;
    return 0;
}
```

Tracing the Recursion

v | 31 | 41 | 59 | 26

elems | 31 | 41 | 59 | 26

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion

v | 31 | 41 | 59 | 26

elems | 31 | 41 | 59 | 26

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
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    return max(maxOf(left), maxOf(right));
  }
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```

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
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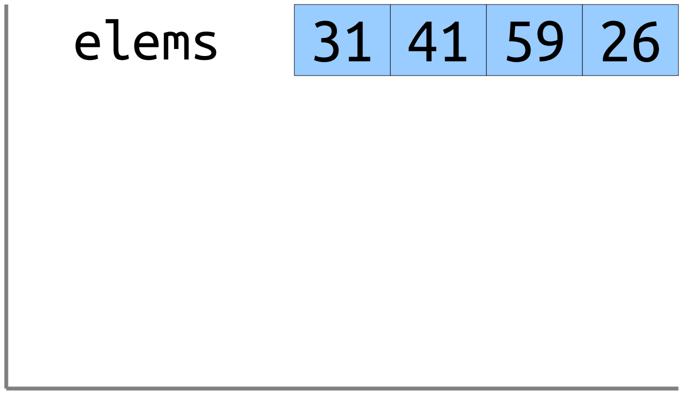
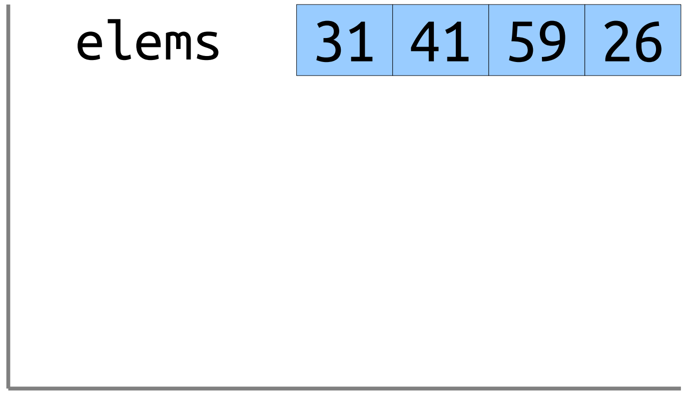
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

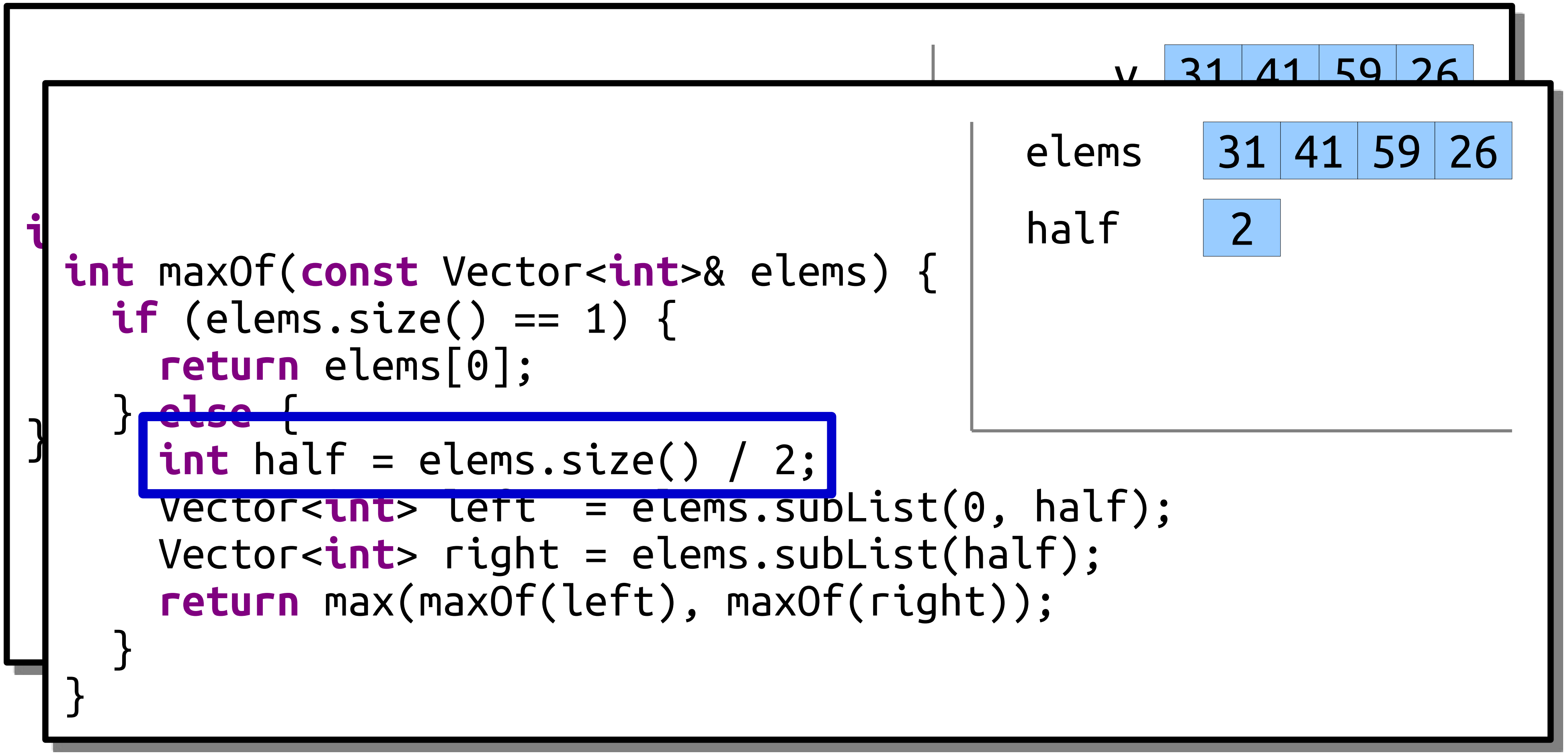


Tracing the Recursion



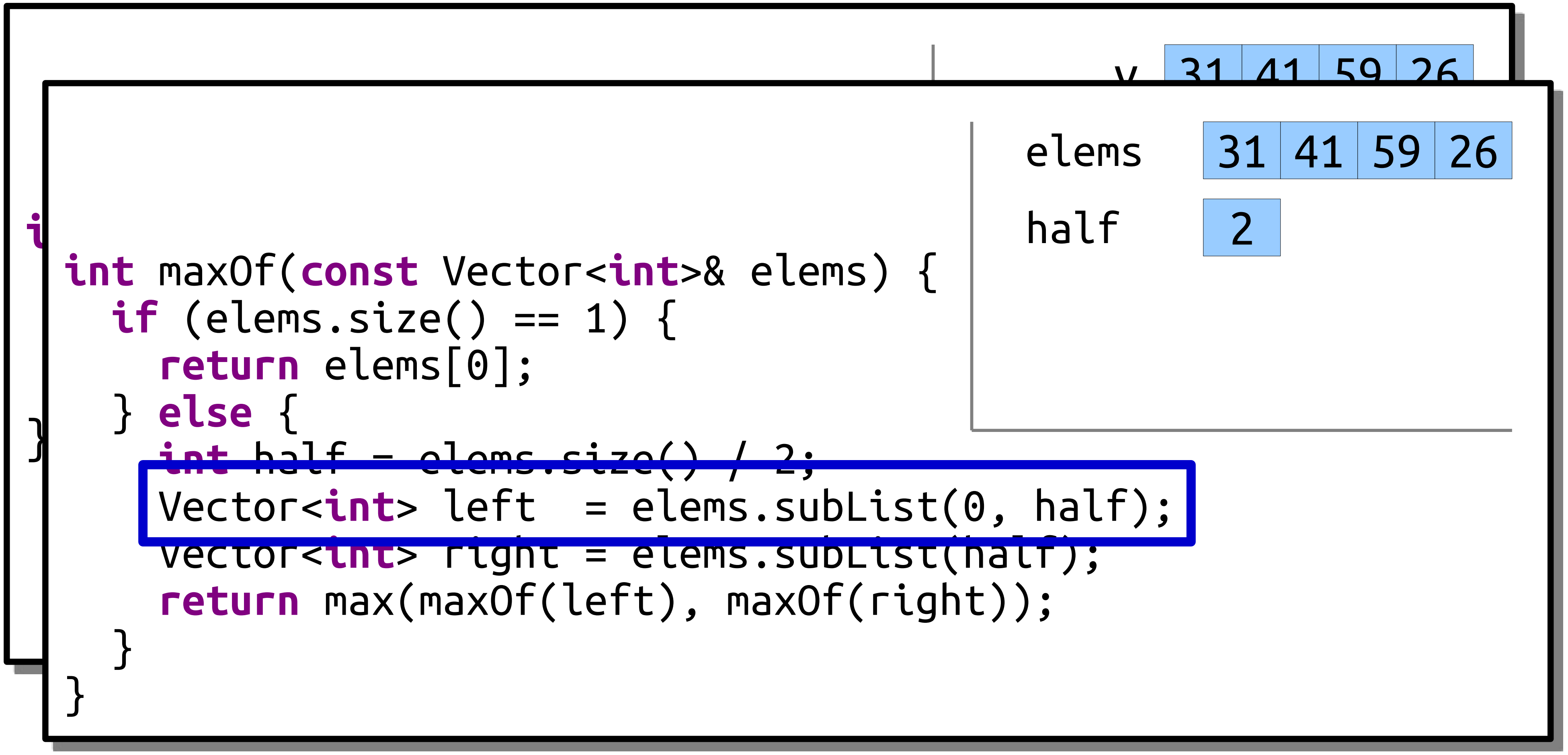
```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

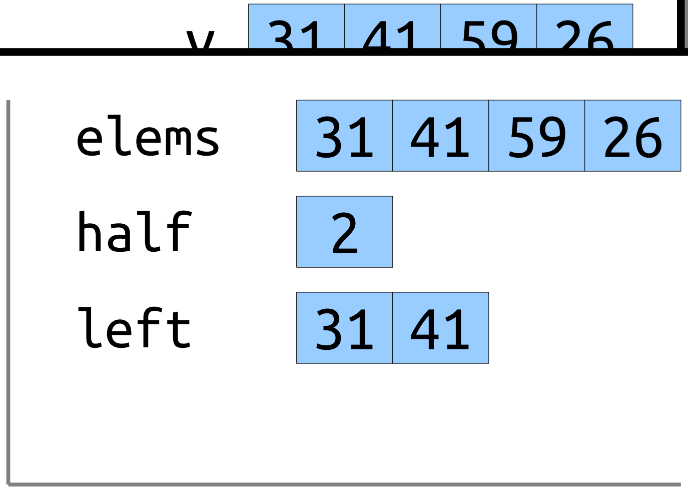
Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

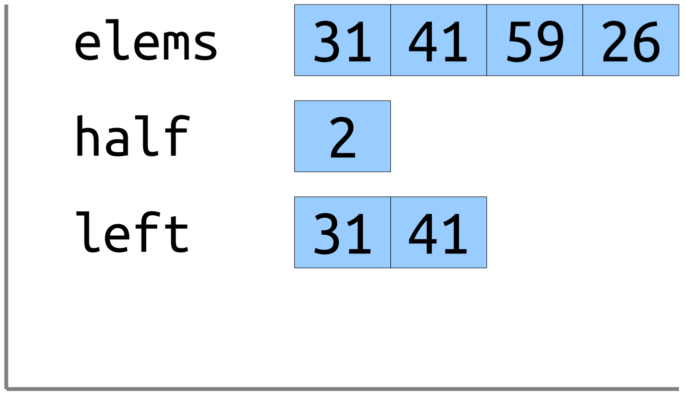
Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```



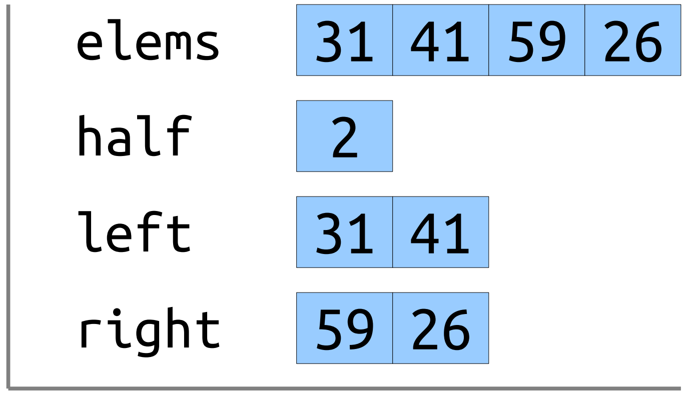
Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```



Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```



elems	31	41	59	26
half	2			
left	31	41		
right	59	26		

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

elems	31	41	59	26
half	2			
left	31	41		
right	59	26		

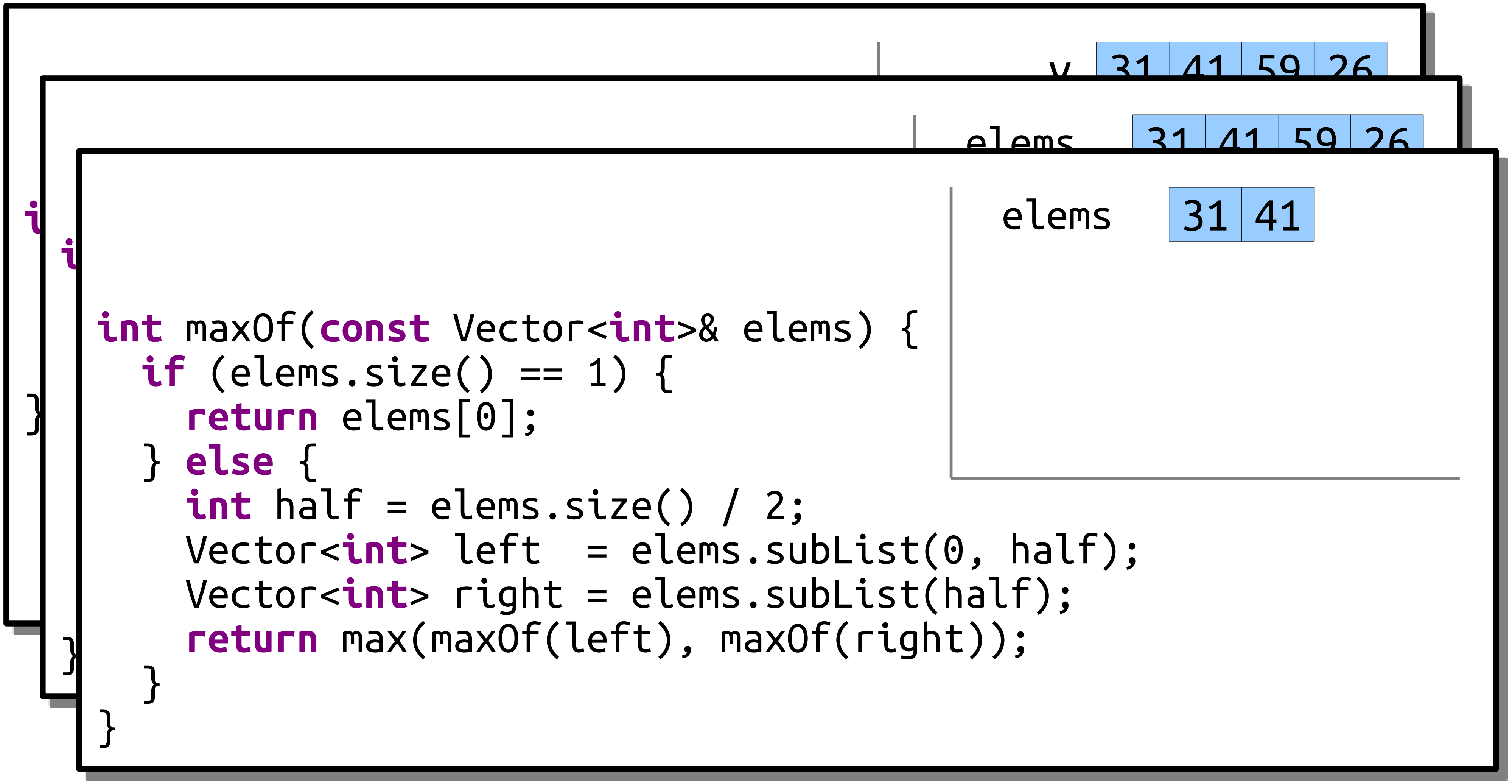
Tracing the Recursion

v 31 41 59 26

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

elems	31	41	59	26
half	2			
left	31	41		
right	59	26		

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion

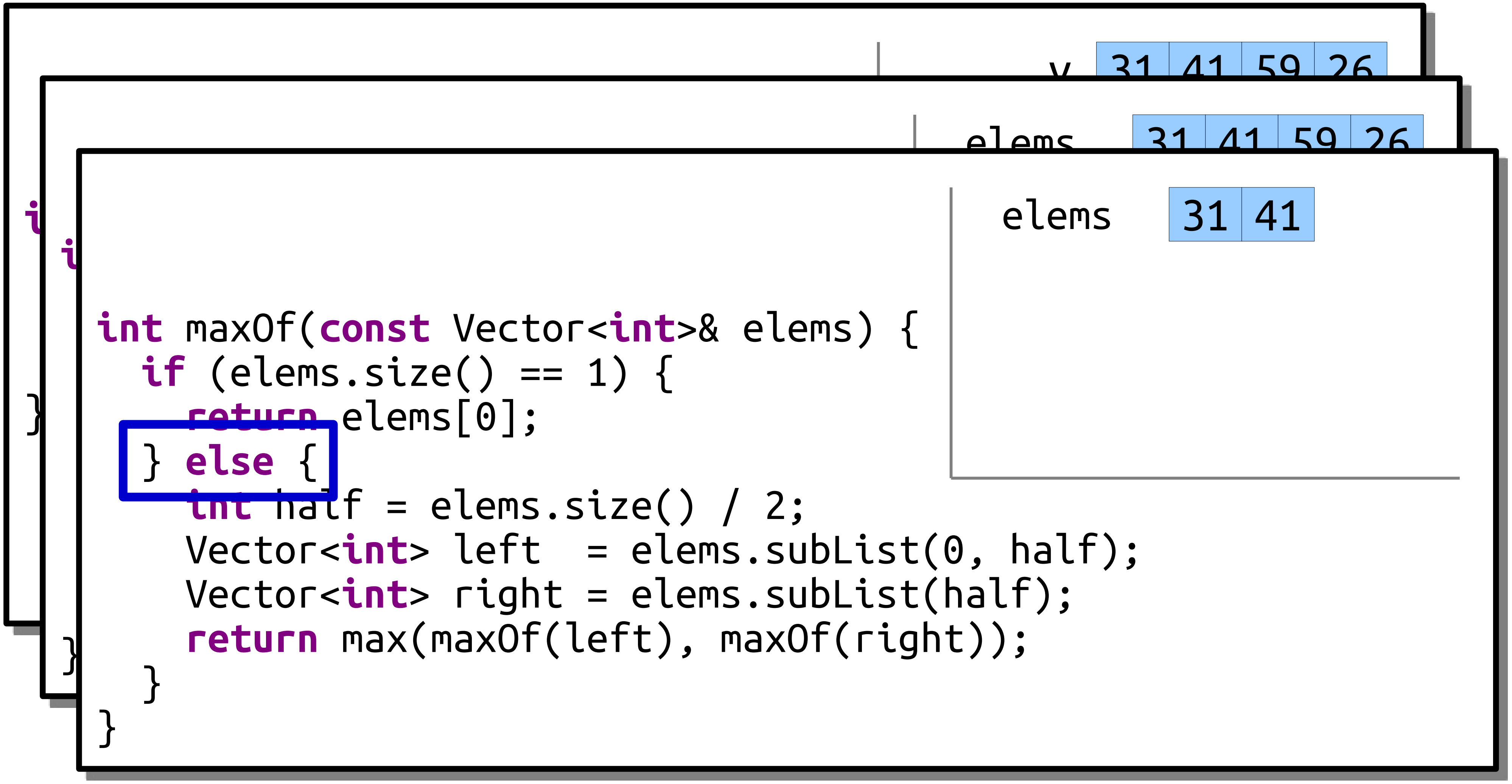
v | 31 | 41 | 59 | 26

elems | 31 | 41 | 59 | 26

elems | 31 | 41

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

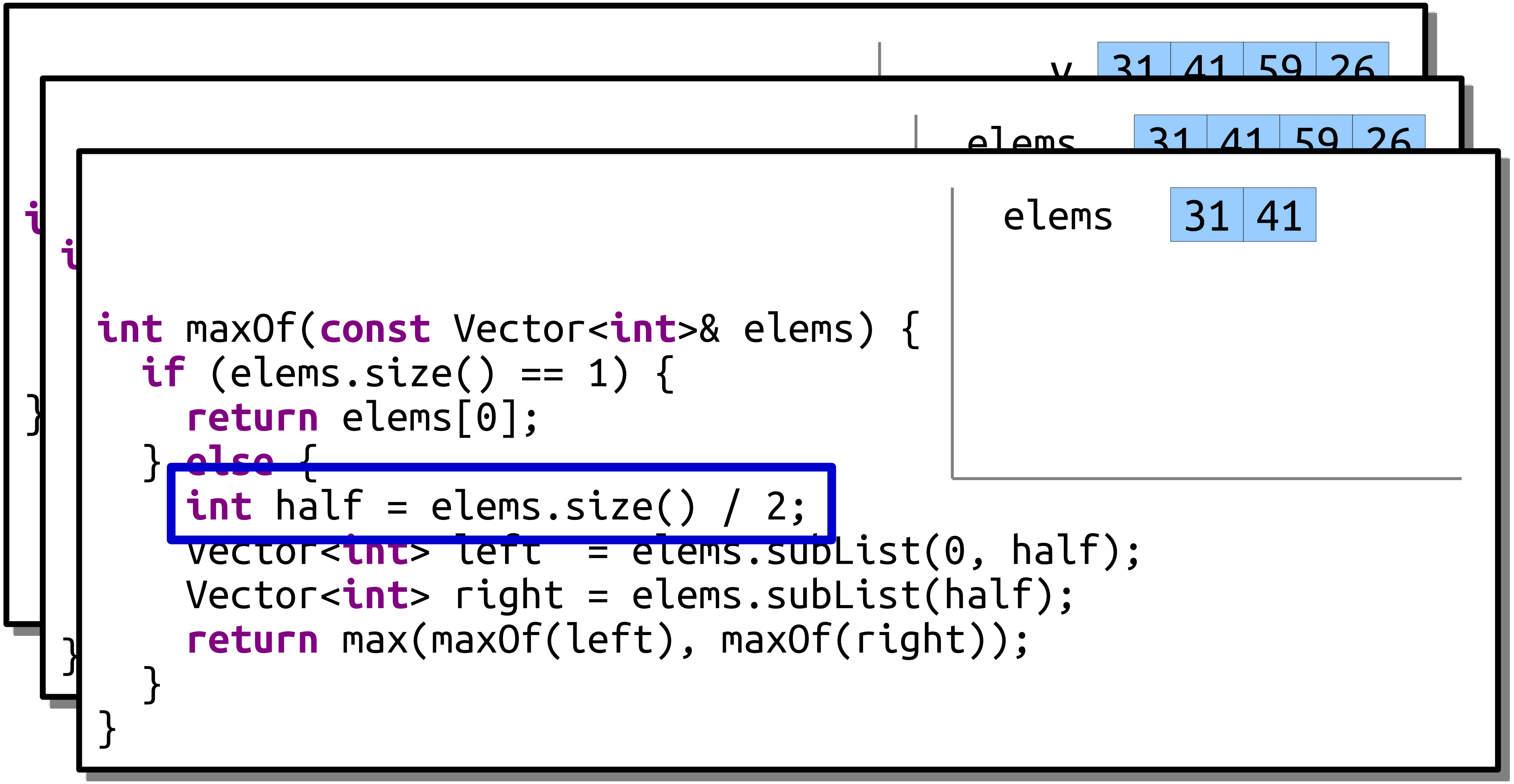
Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

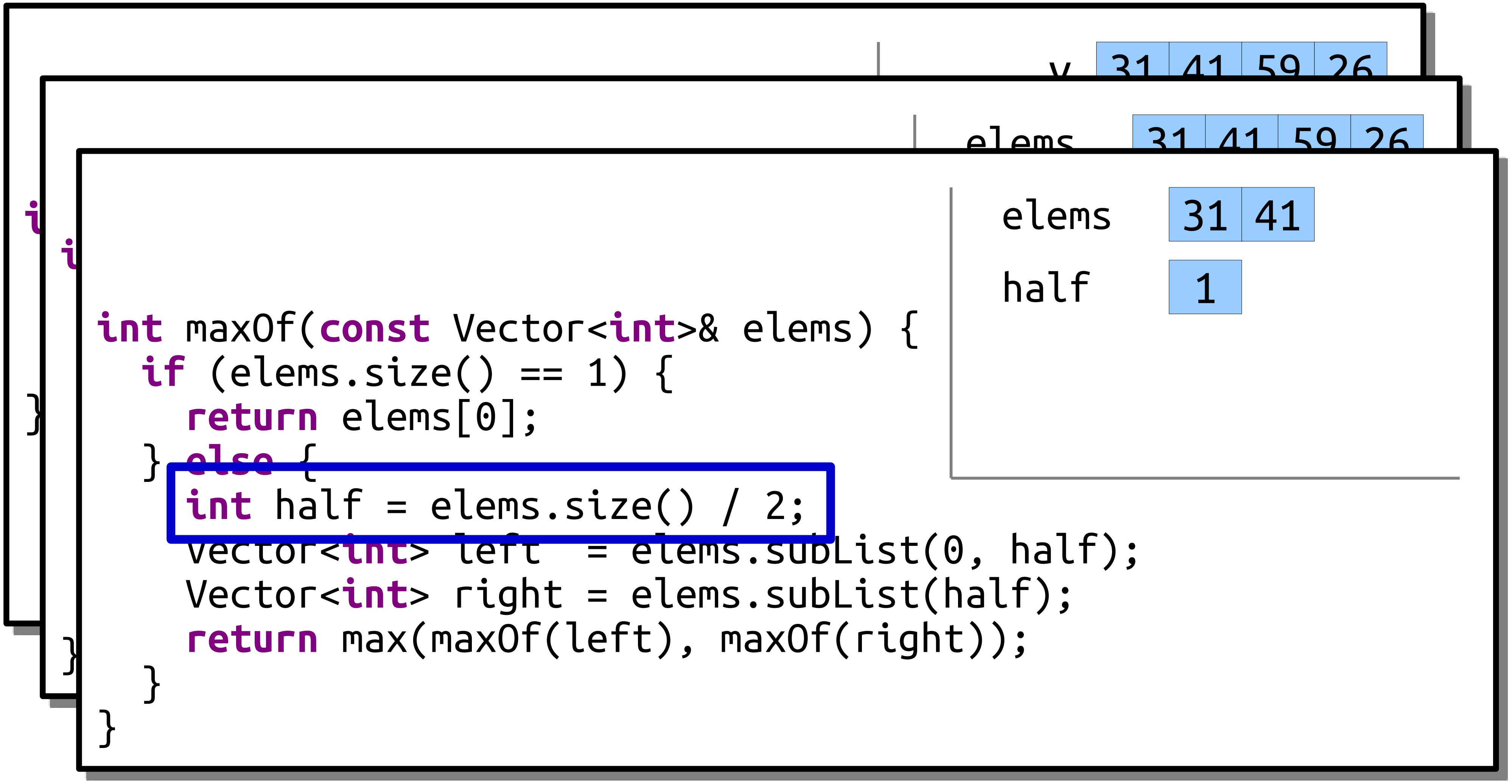
elems	31	41

Tracing the Recursion



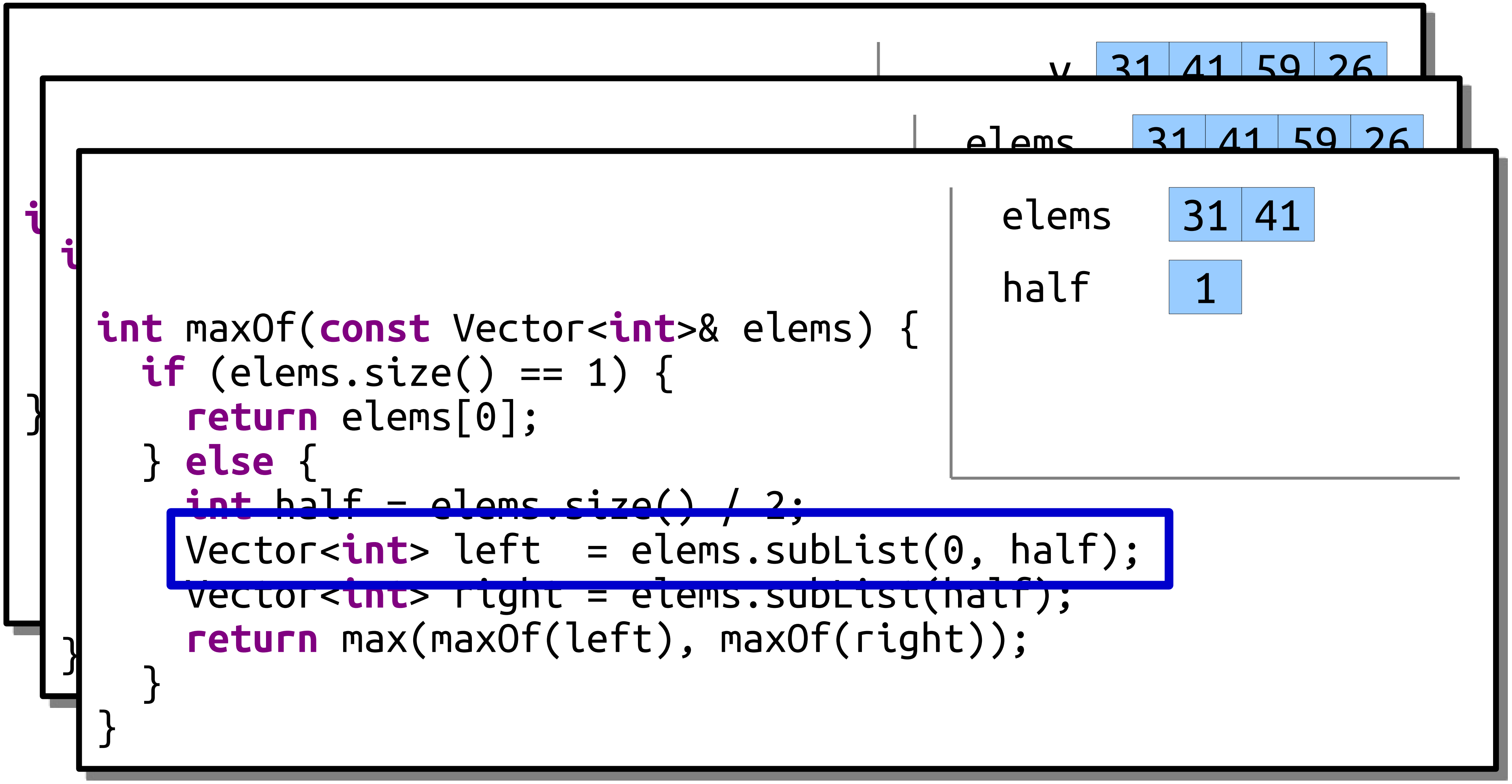
```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



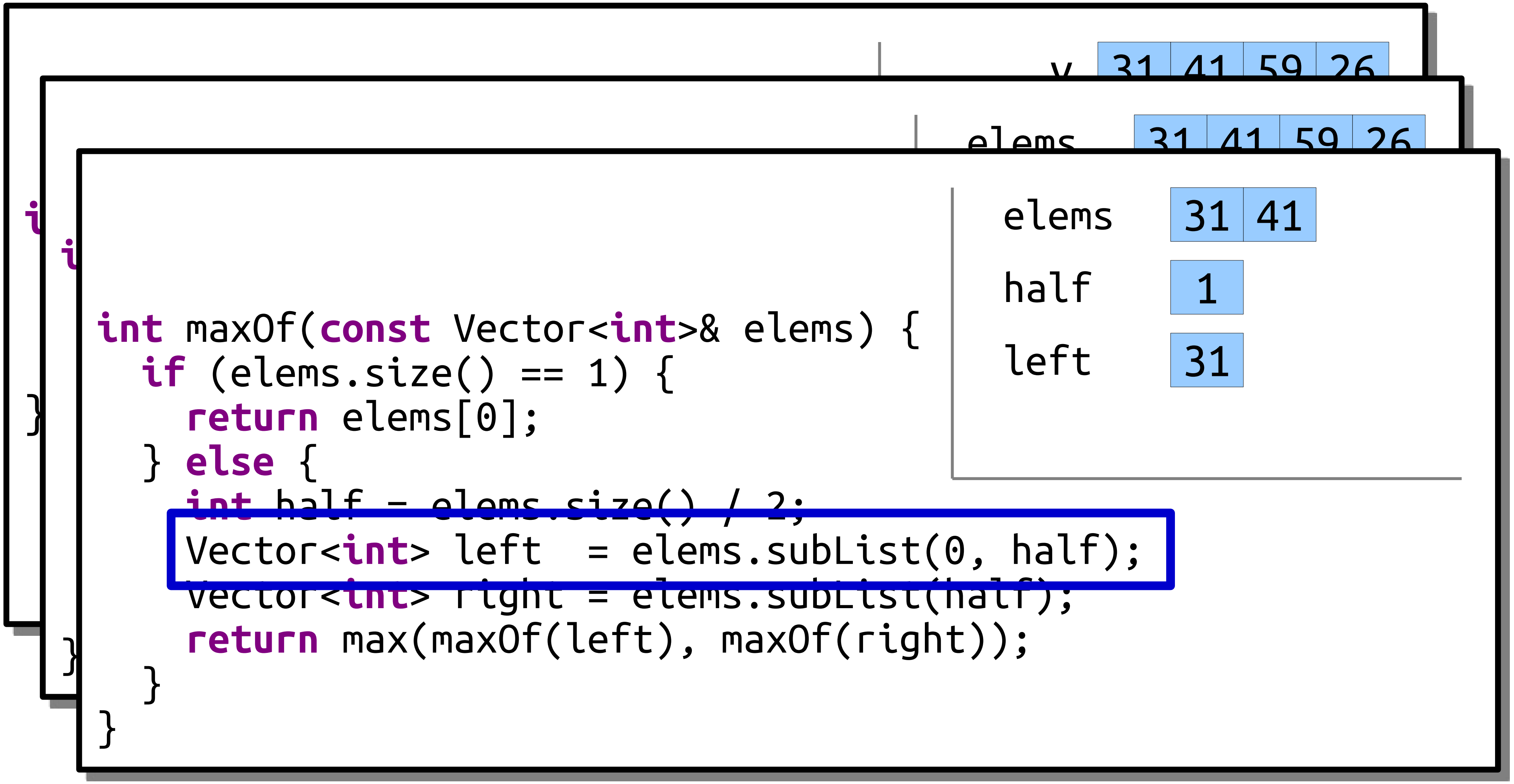
```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

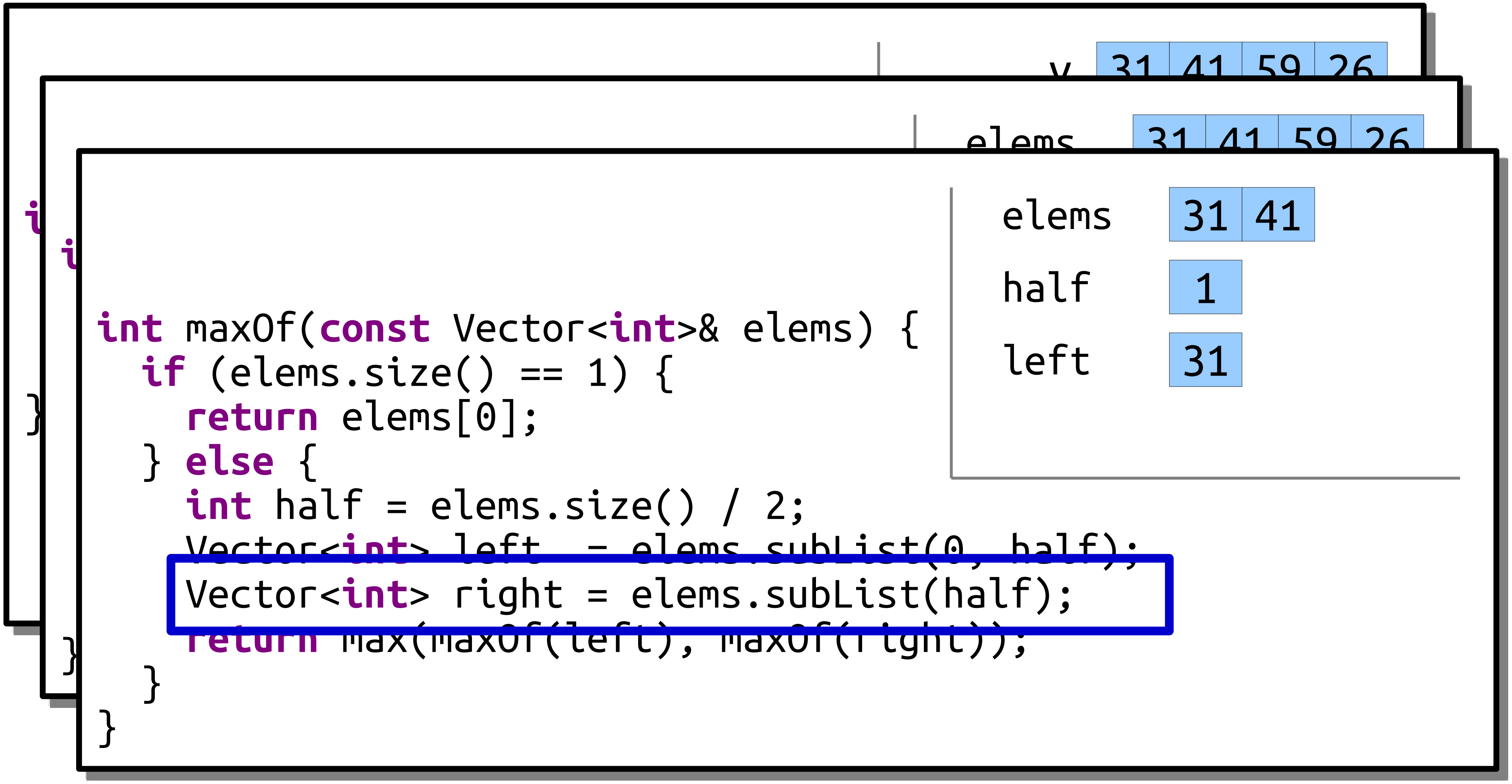
Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

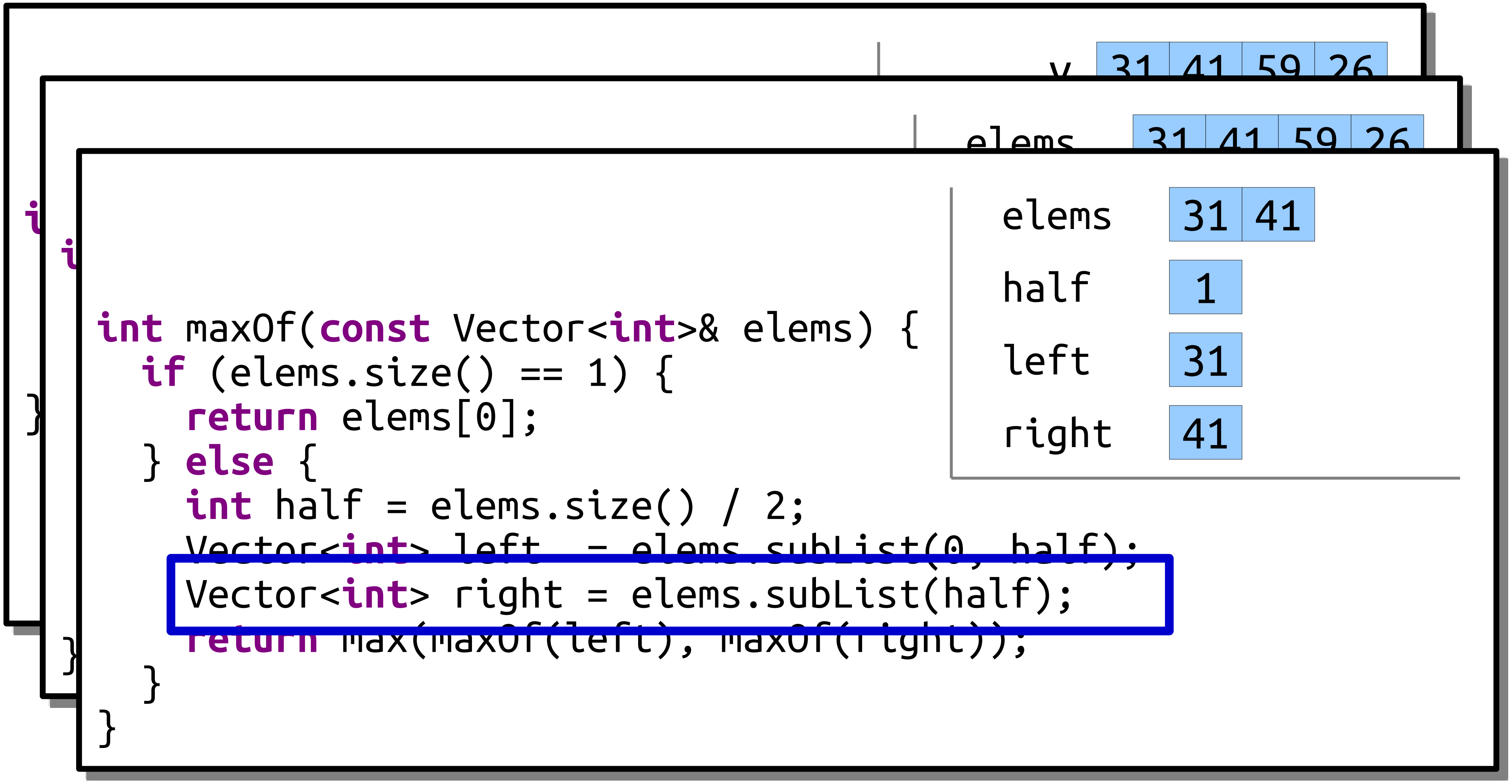
elems	31	41
half	1	
left	31	

Tracing the Recursion



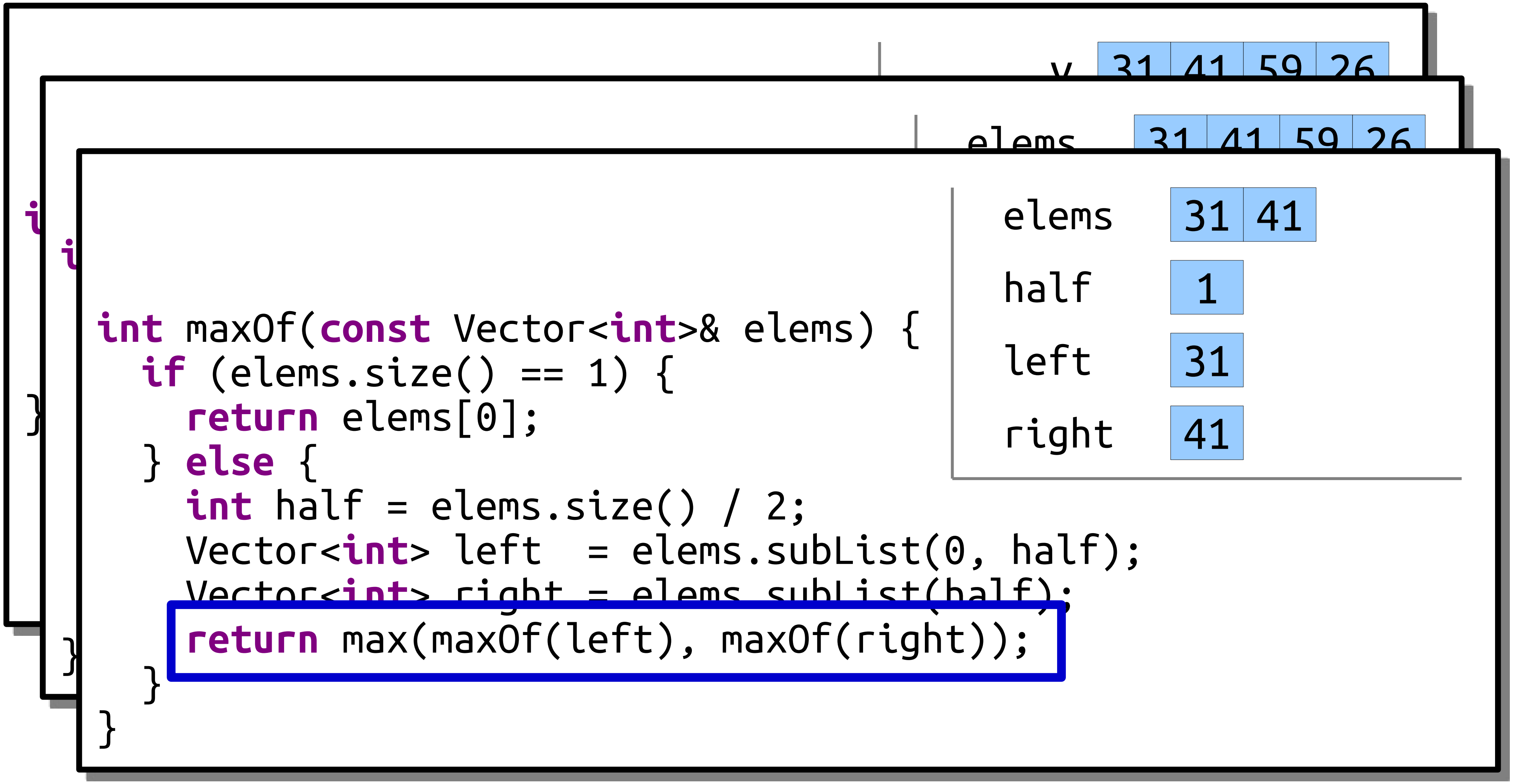
```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

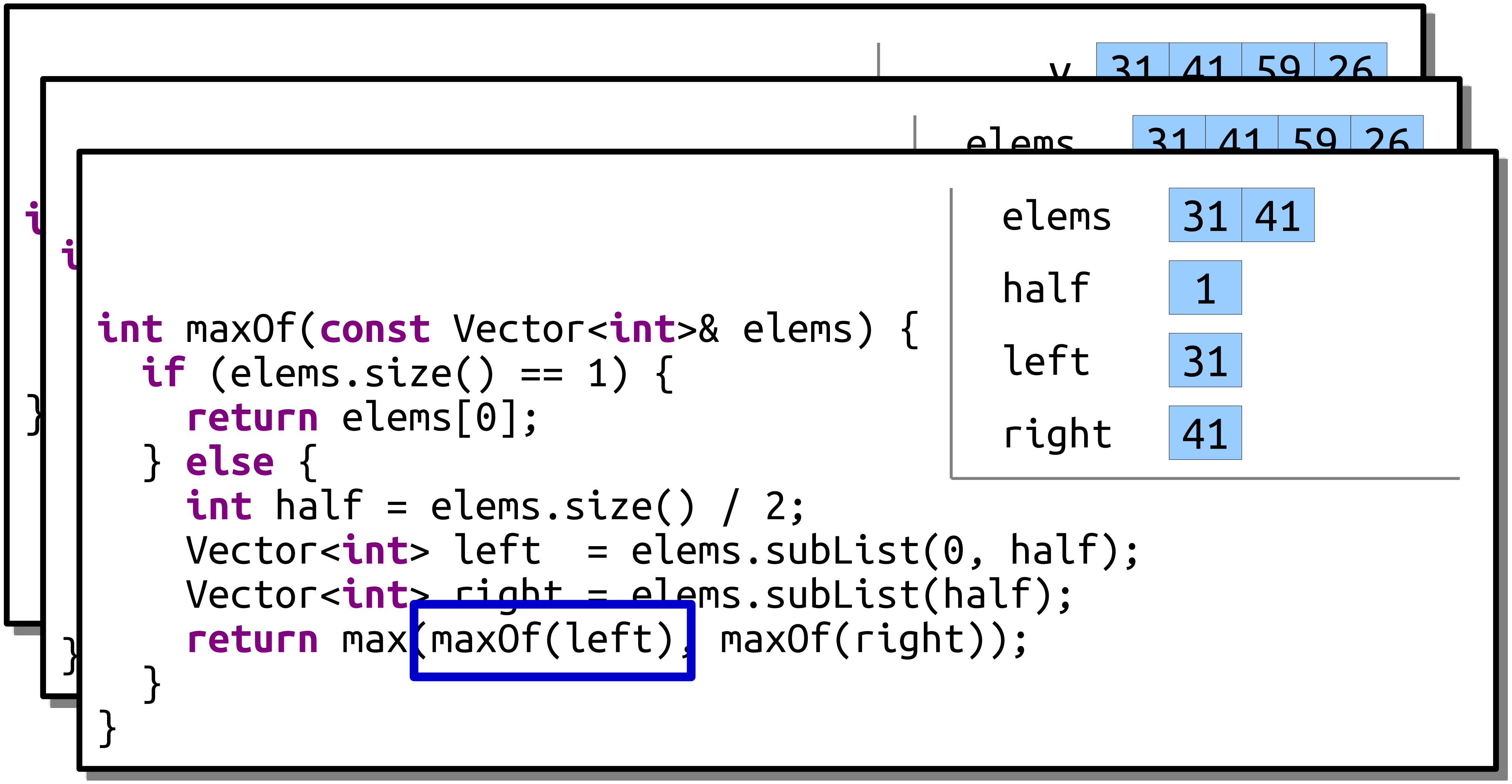
Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

elems	31 41
half	1
left	31
right	41

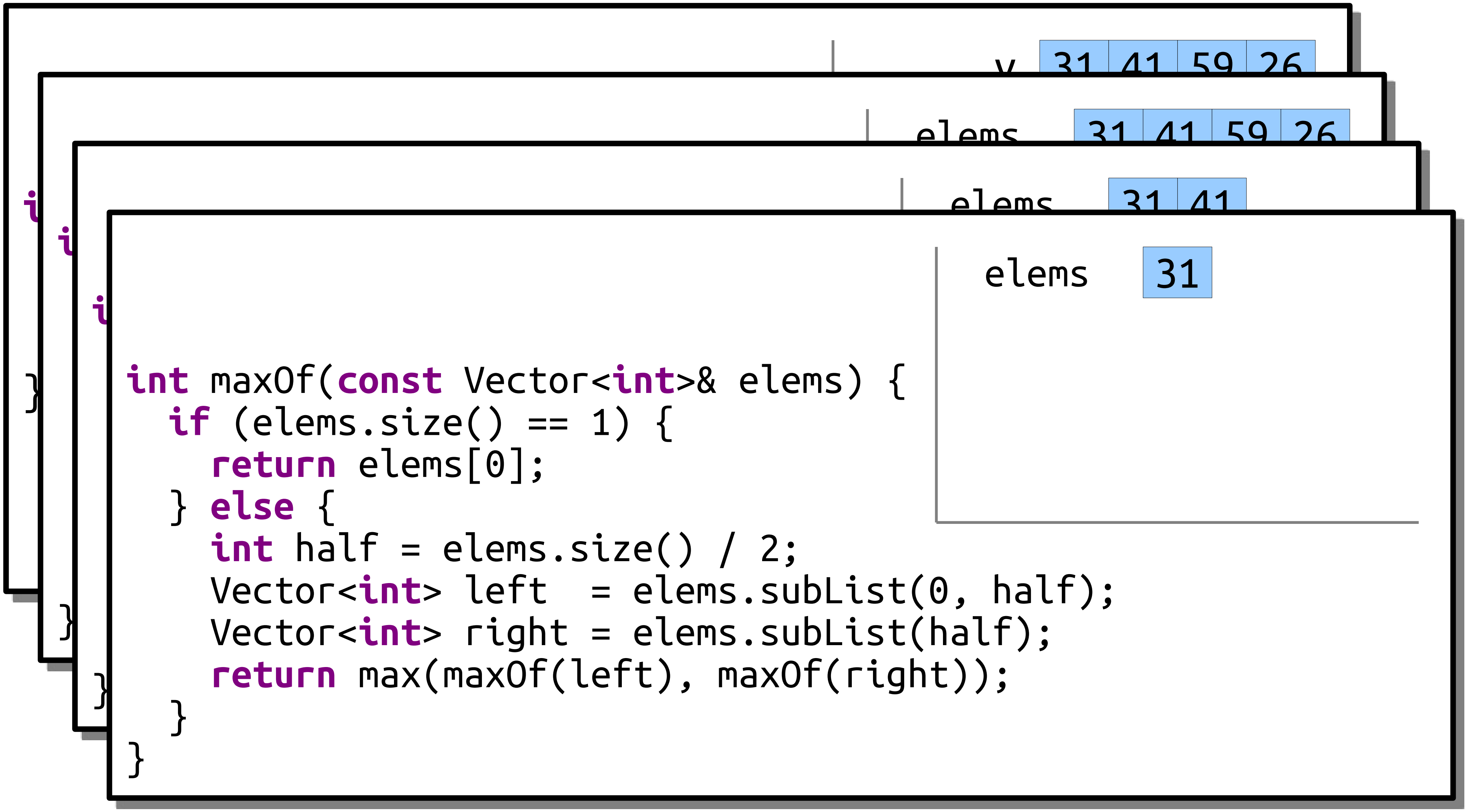
Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

elems	31	41
half	1	
left	31	
right	41	

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

elems	31

Tracing the Recursion

v 31 41 59 26

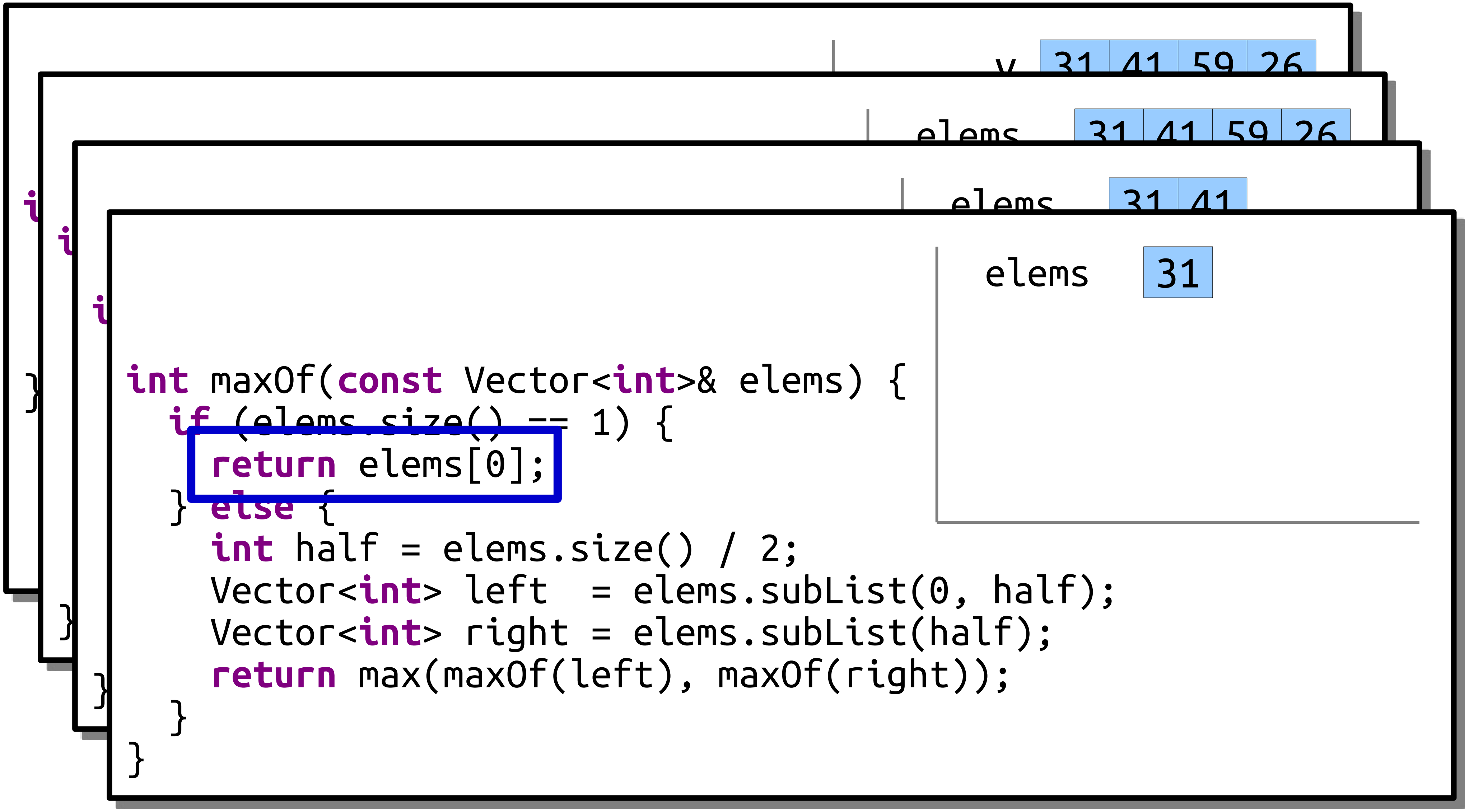
elems 31 41 59 26

elems 31 41

elems 31

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
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  } else {
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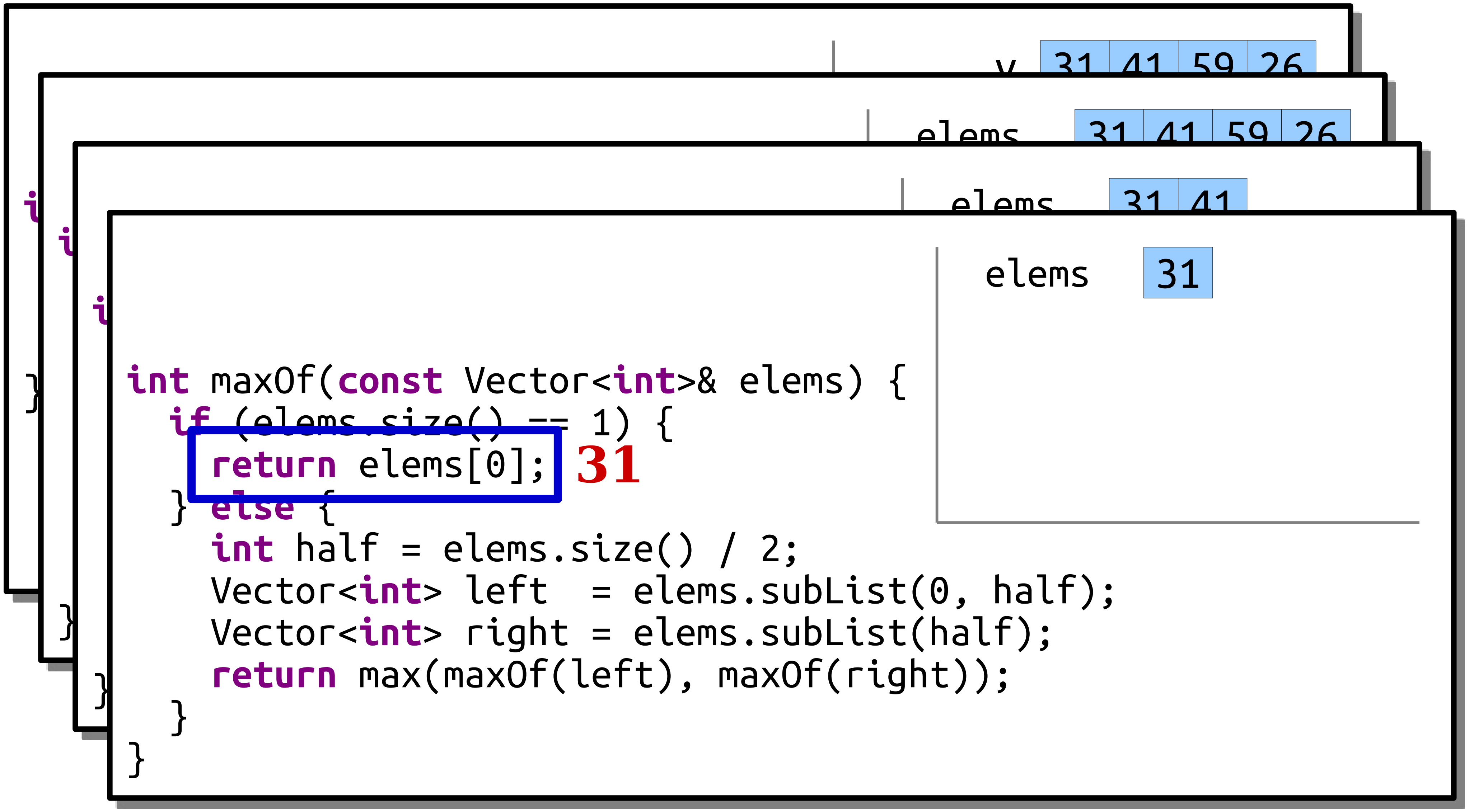
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
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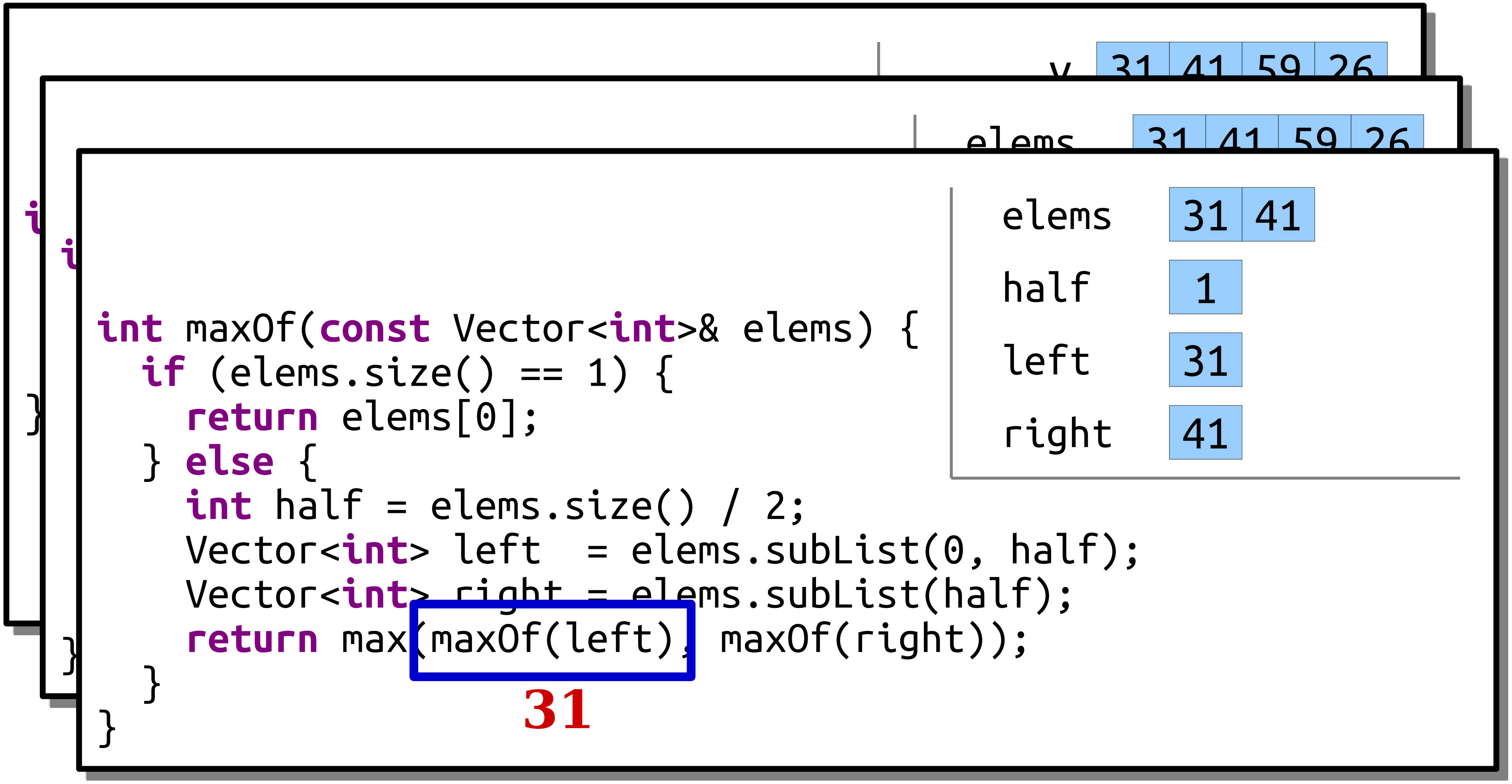
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion

v	31	41	59	26

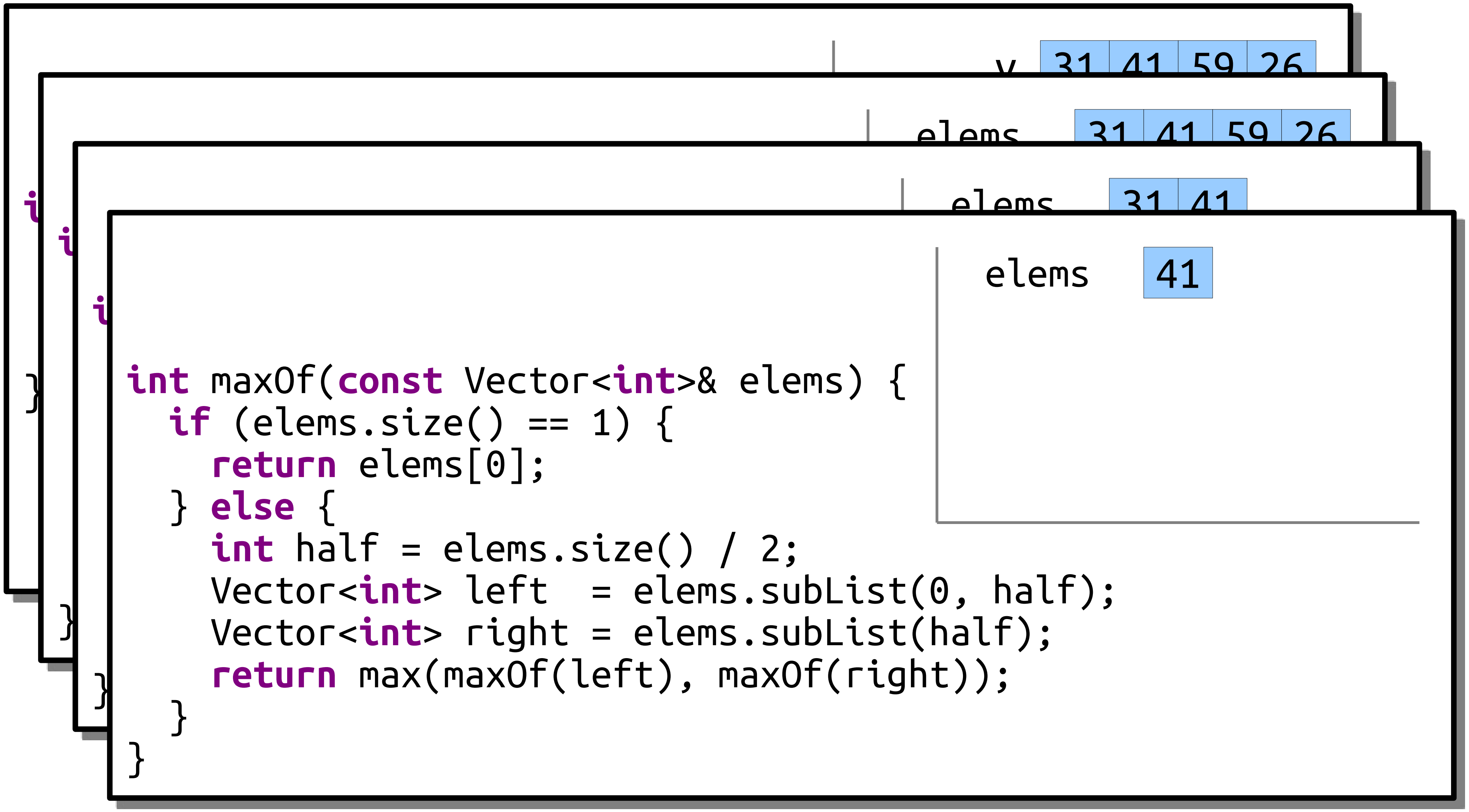
elems	31	41	59	26

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

31

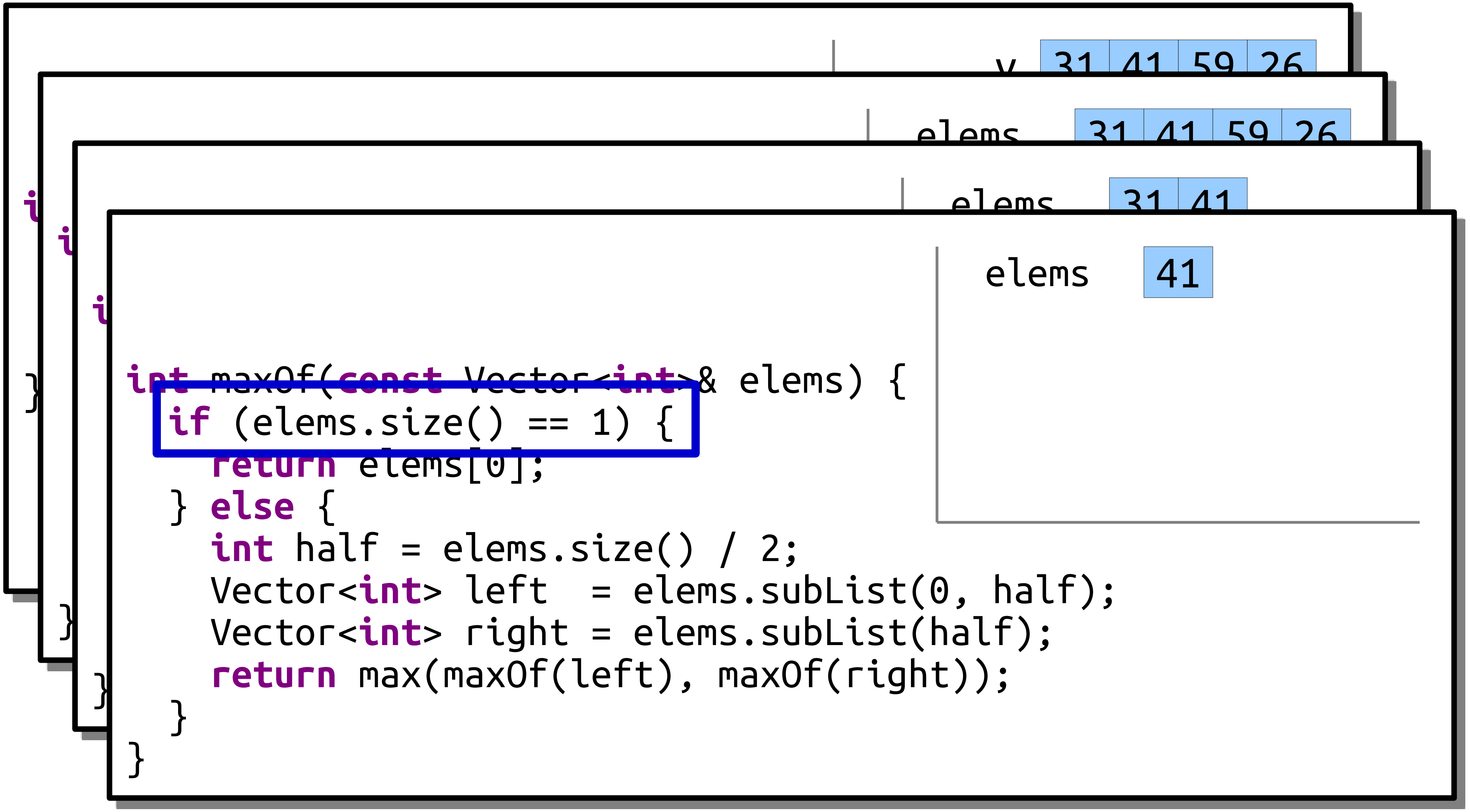
elems	31	41
half	1	
left	31	
right	41	

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion

v 31 41 59 26

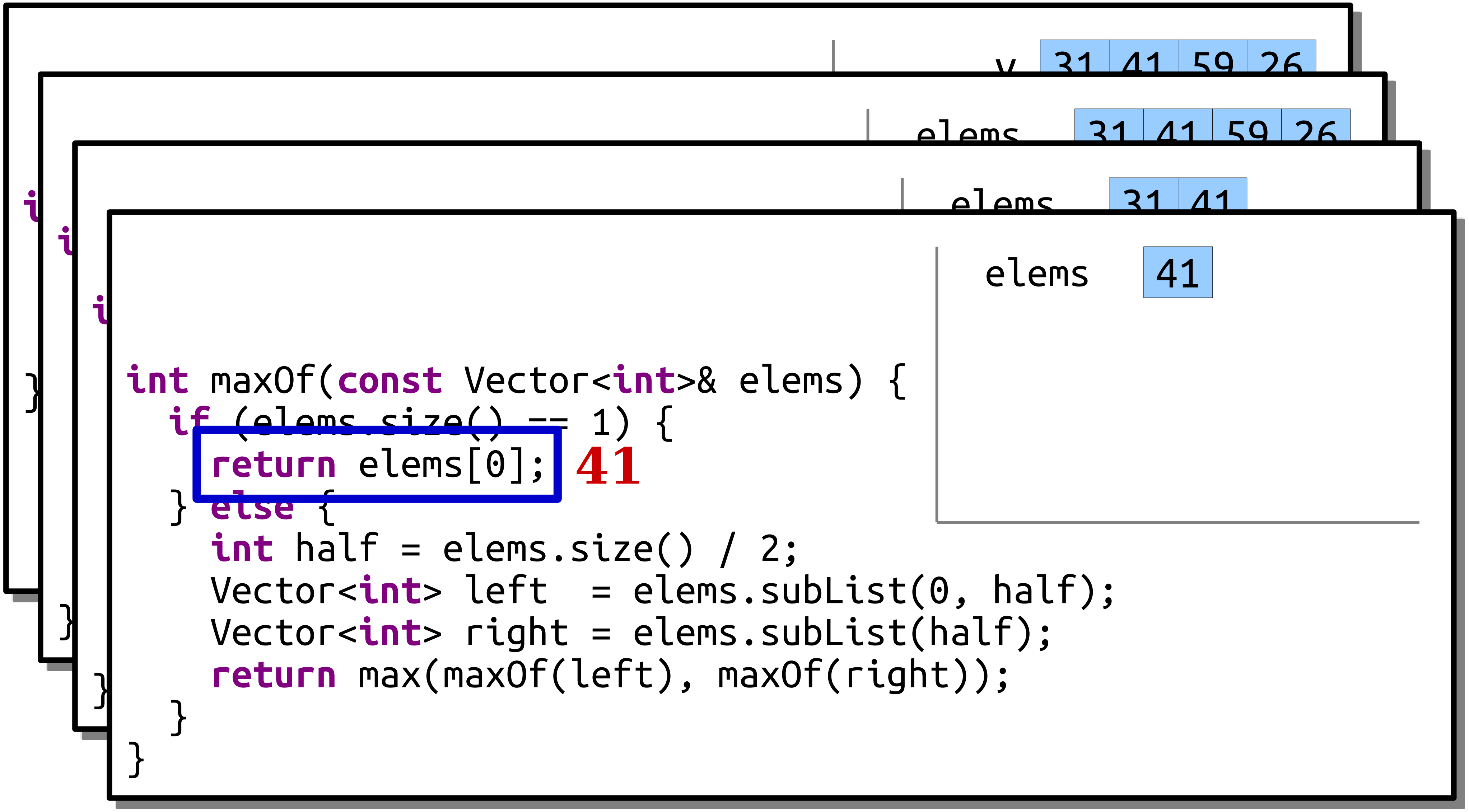
elems 31 41 59 26

elems 31 41

elems 41

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];  41
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion

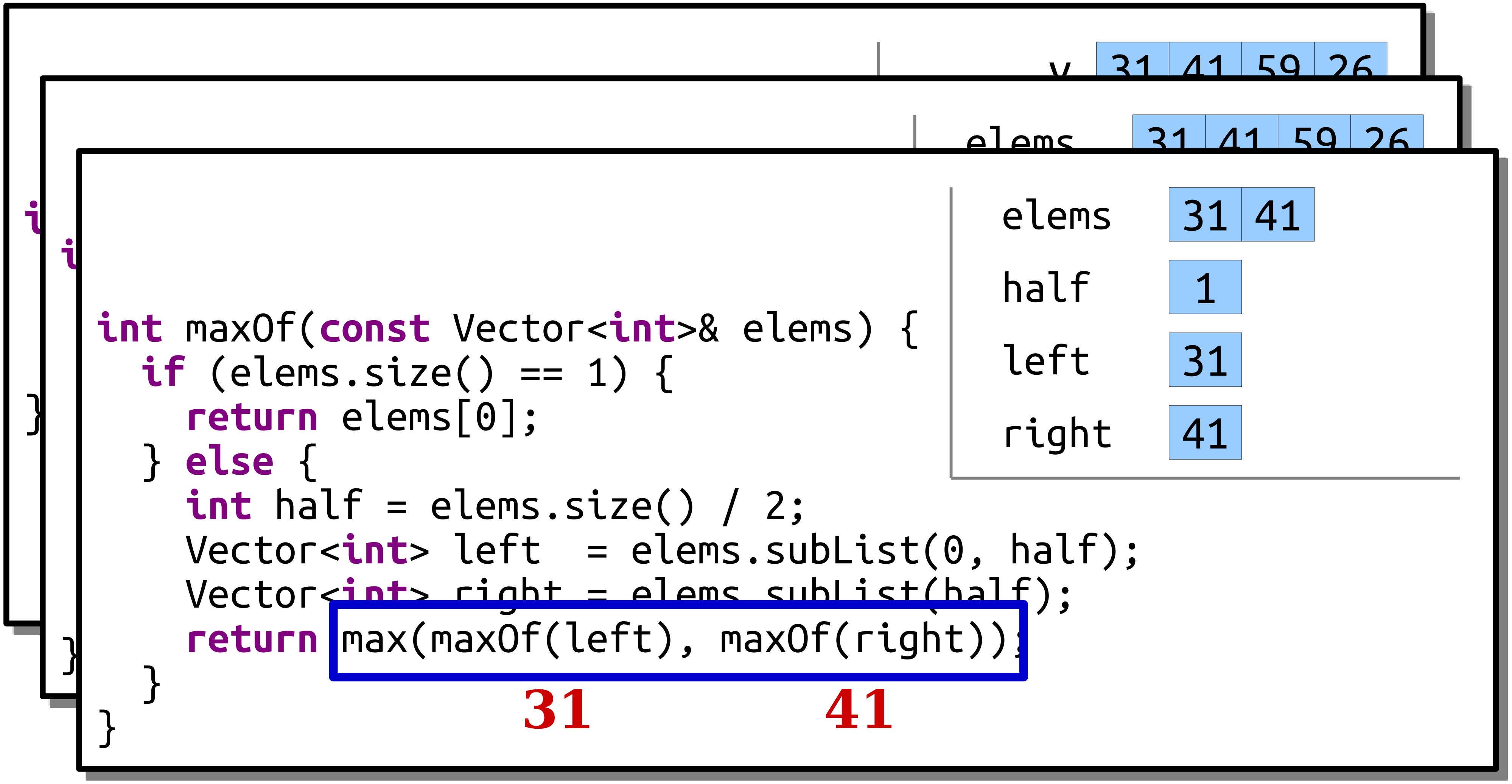
v	31	41	59	26
elems	31	41	59	26

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

31 41

elems	31	41
half	1	
left	31	
right	41	

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

$$\boxed{\texttt{max(maxOf(left), maxOf(right));}}$$

Tracing the Recursion

v	31	41	59	26

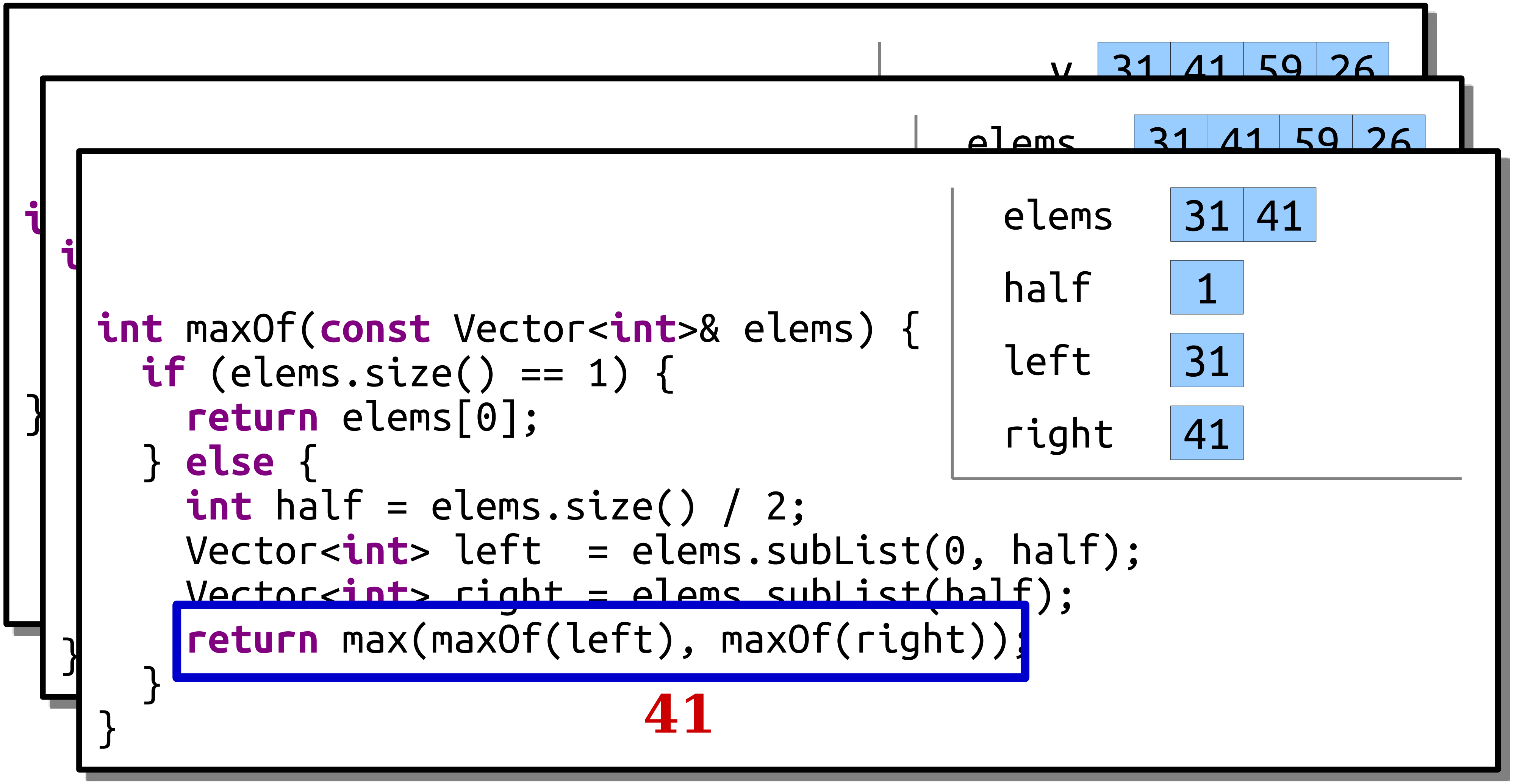
elems	31	41	59	26

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

elems	31	41
half	1	
left	31	
right	41	

41

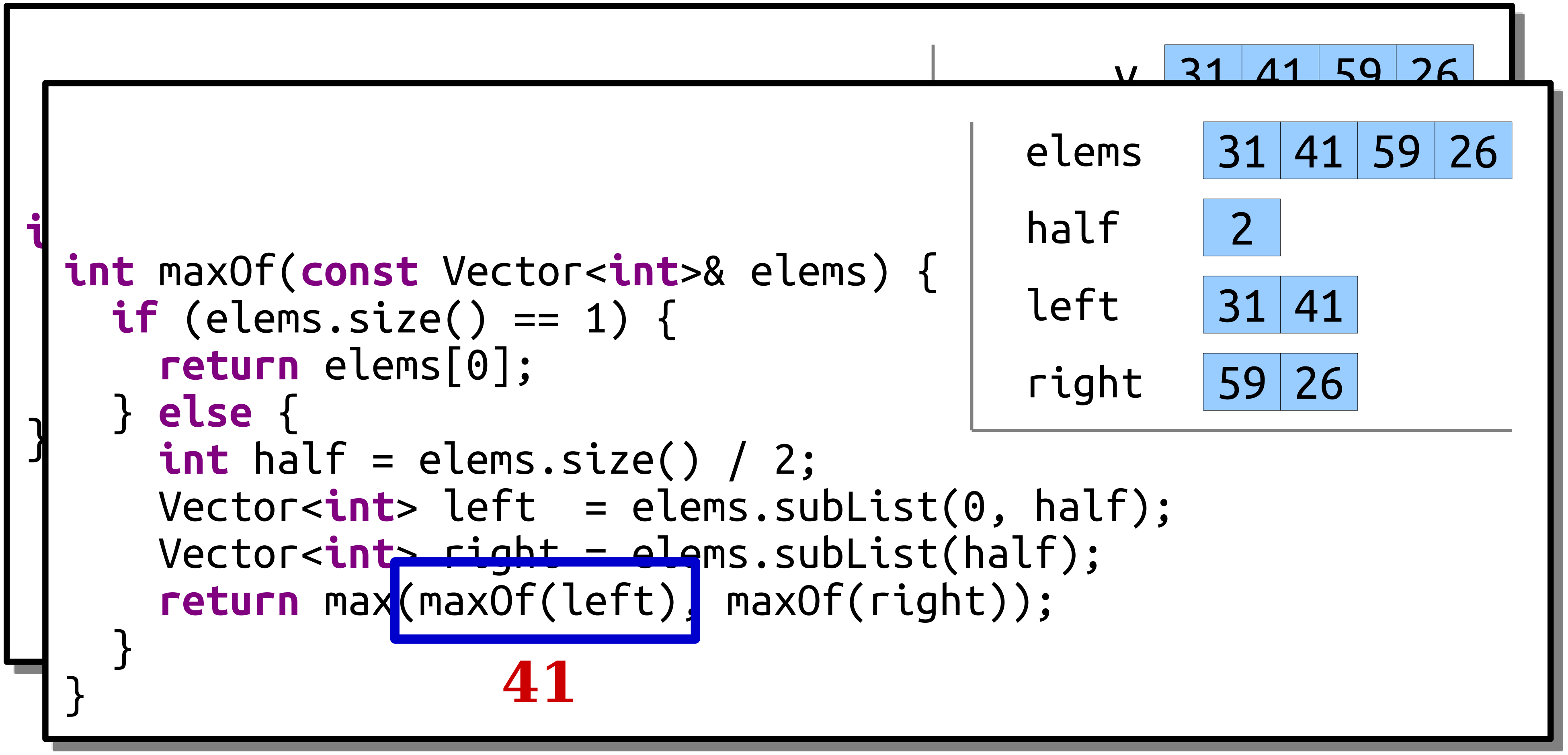
Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

elems	31 41
half	1
left	31
right	41

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

41

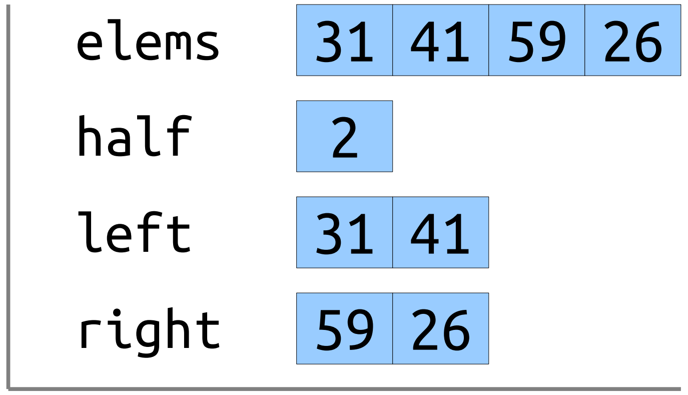
v	31	41	59	26

elems	31	41	59	26
half	2			
left	31	41		
right	59	26		

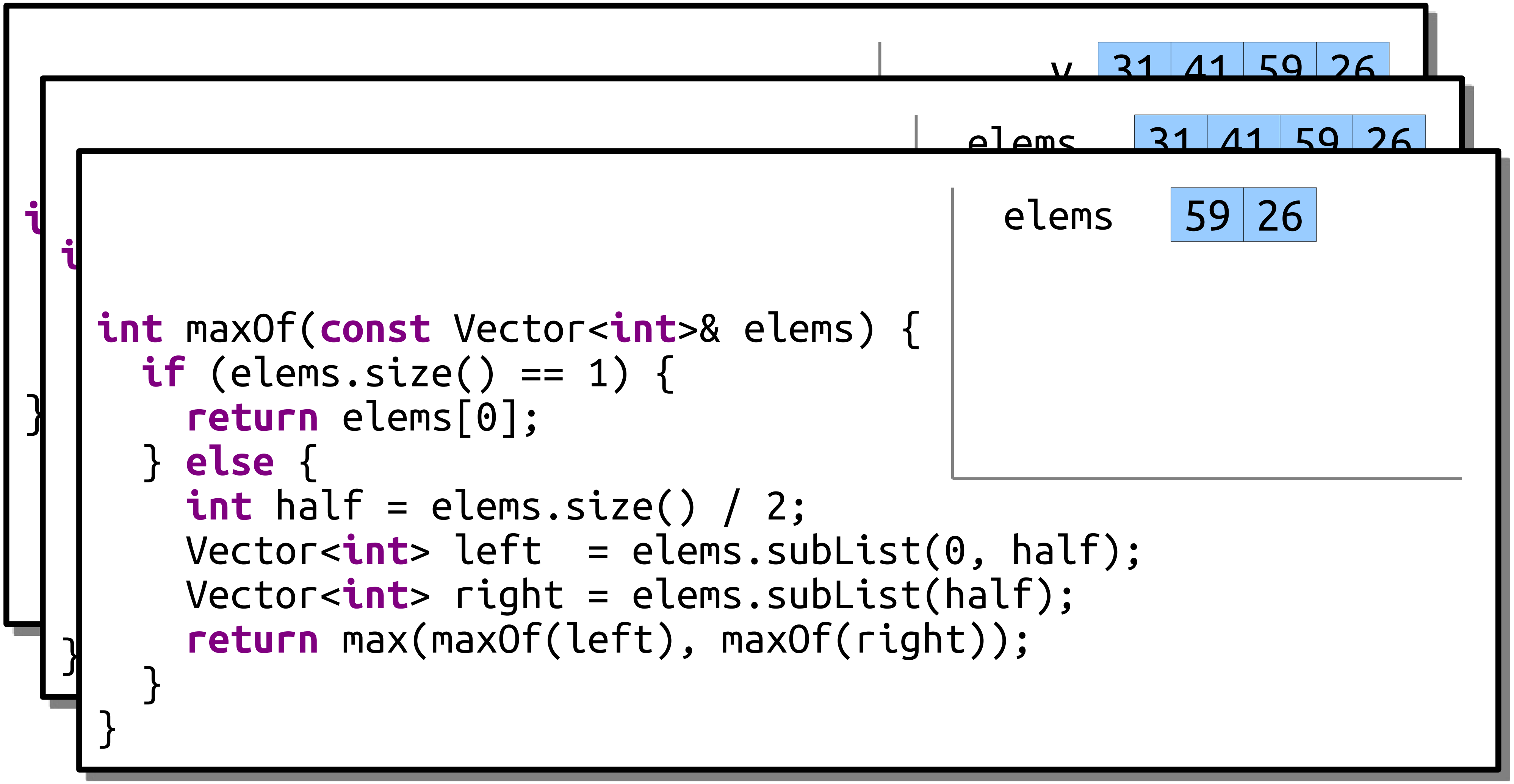
Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

41



Tracing the Recursion



```
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  if (elems.size() == 1) {
    return elems[0];
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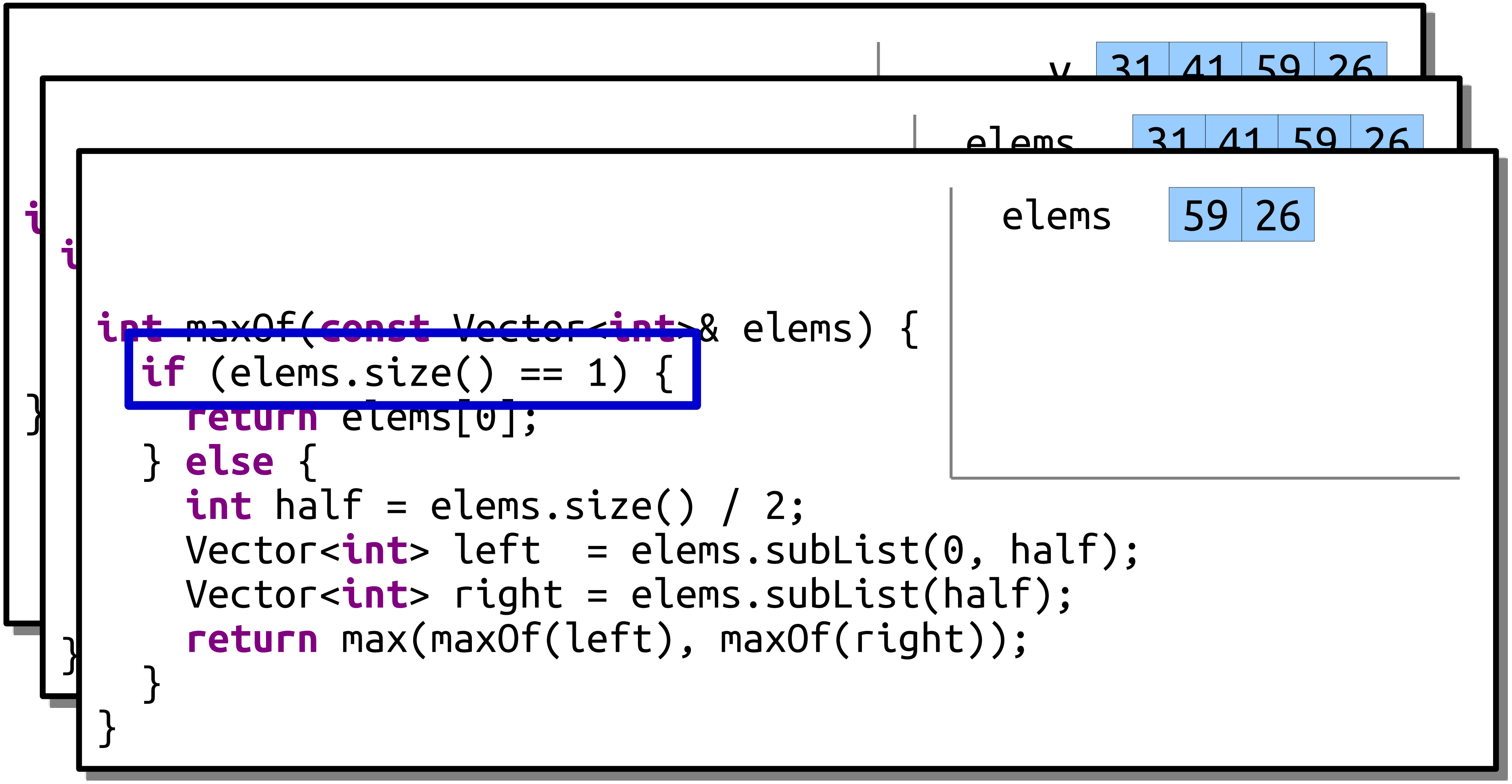
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
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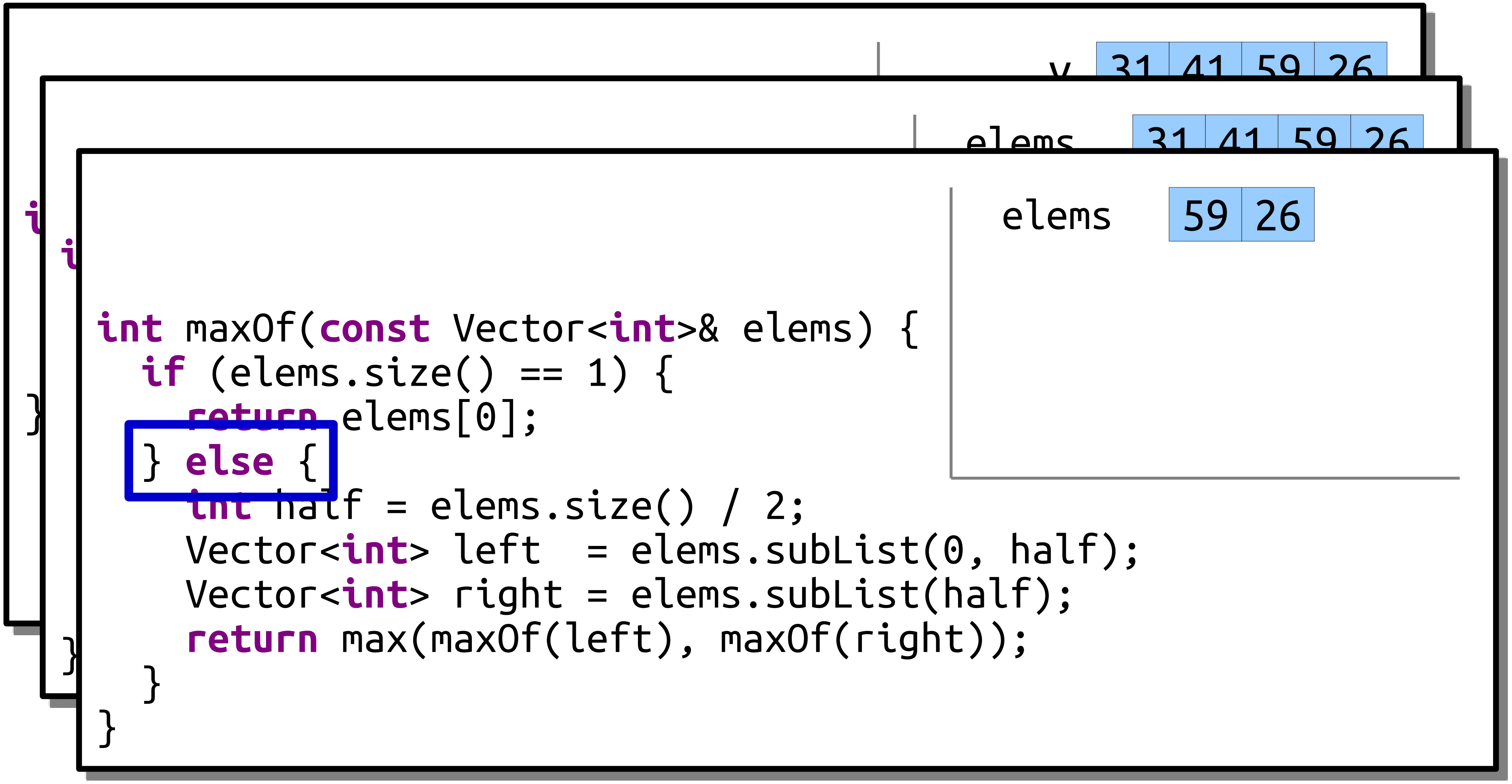
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion

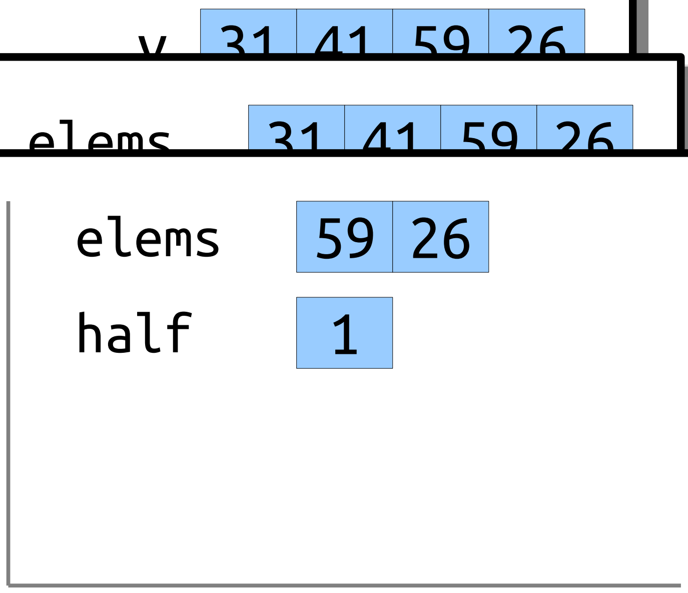
v | 31 | 41 | 59 | 26

elems | 31 | 41 | 59 | 26

elems | 59 | 26

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion

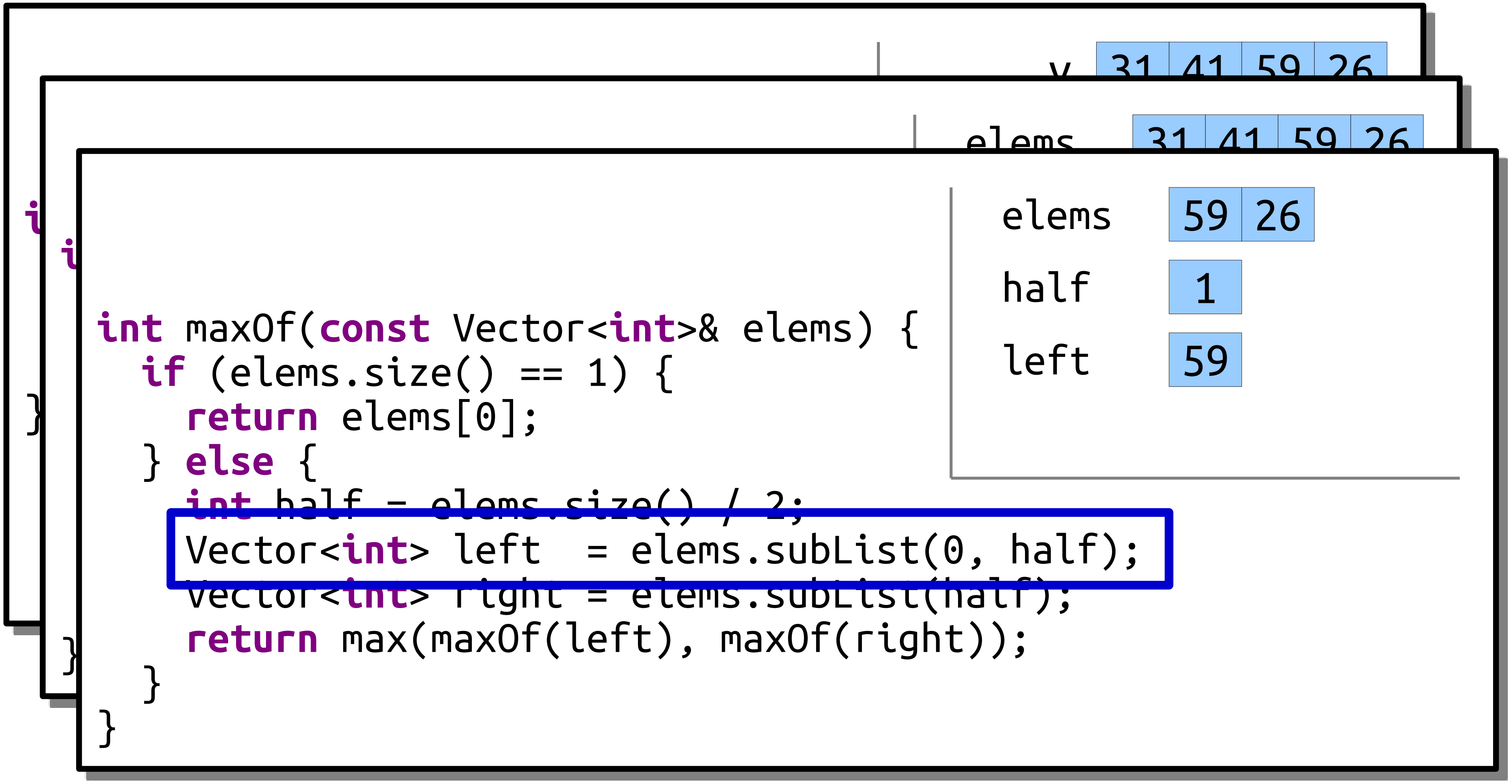
v	31	41	59	26

elems	31	41	59	26

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

elems	59	26
half	1	

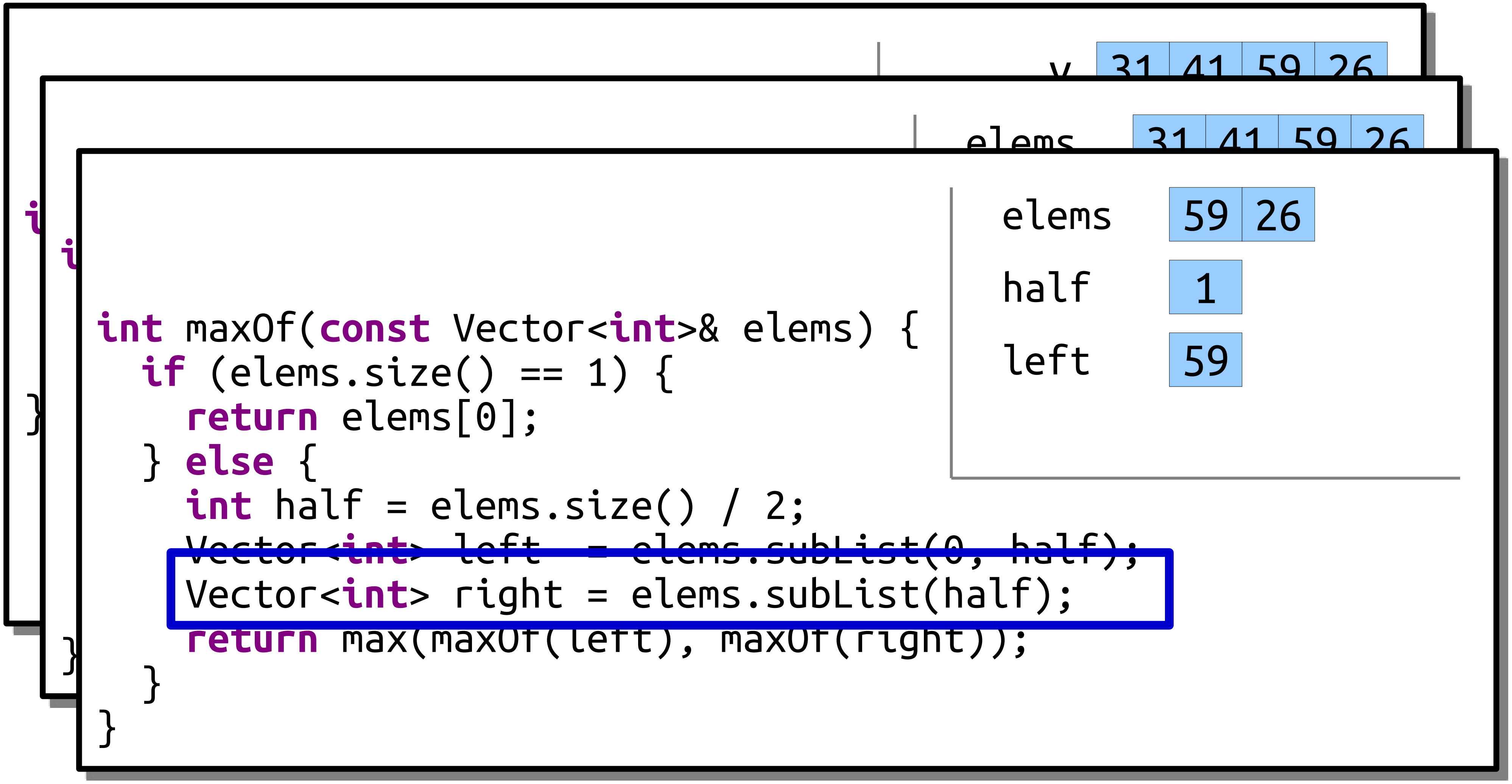
Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

elems	59 26
half	1
left	59

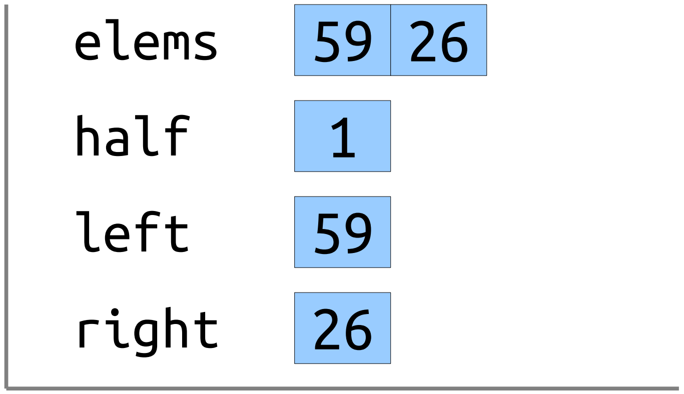
Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```



Tracing the Recursion

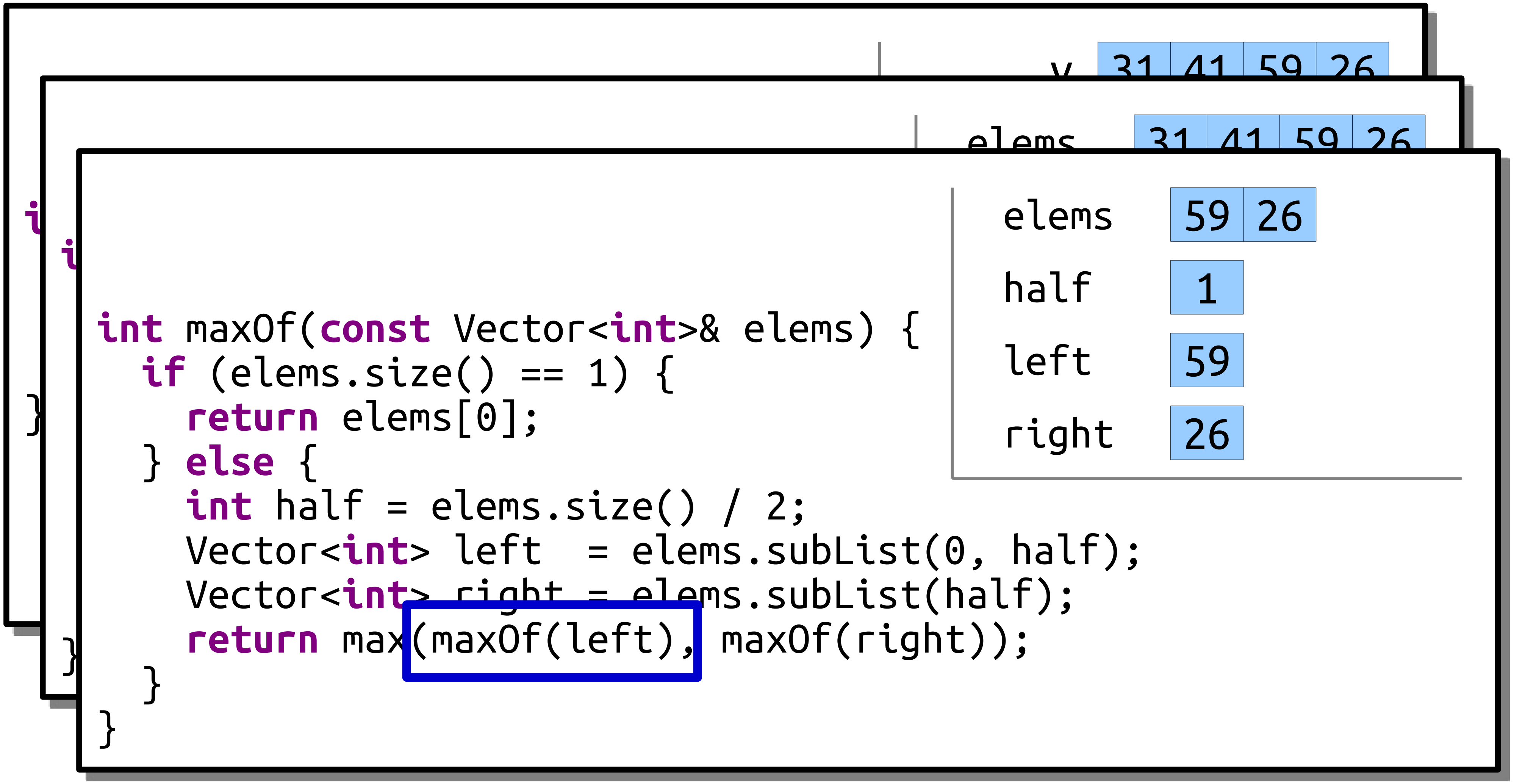
v	31	41	59	26

elems	31	41	59	26

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

elems	59	26
half	1	
left	59	
right	26	

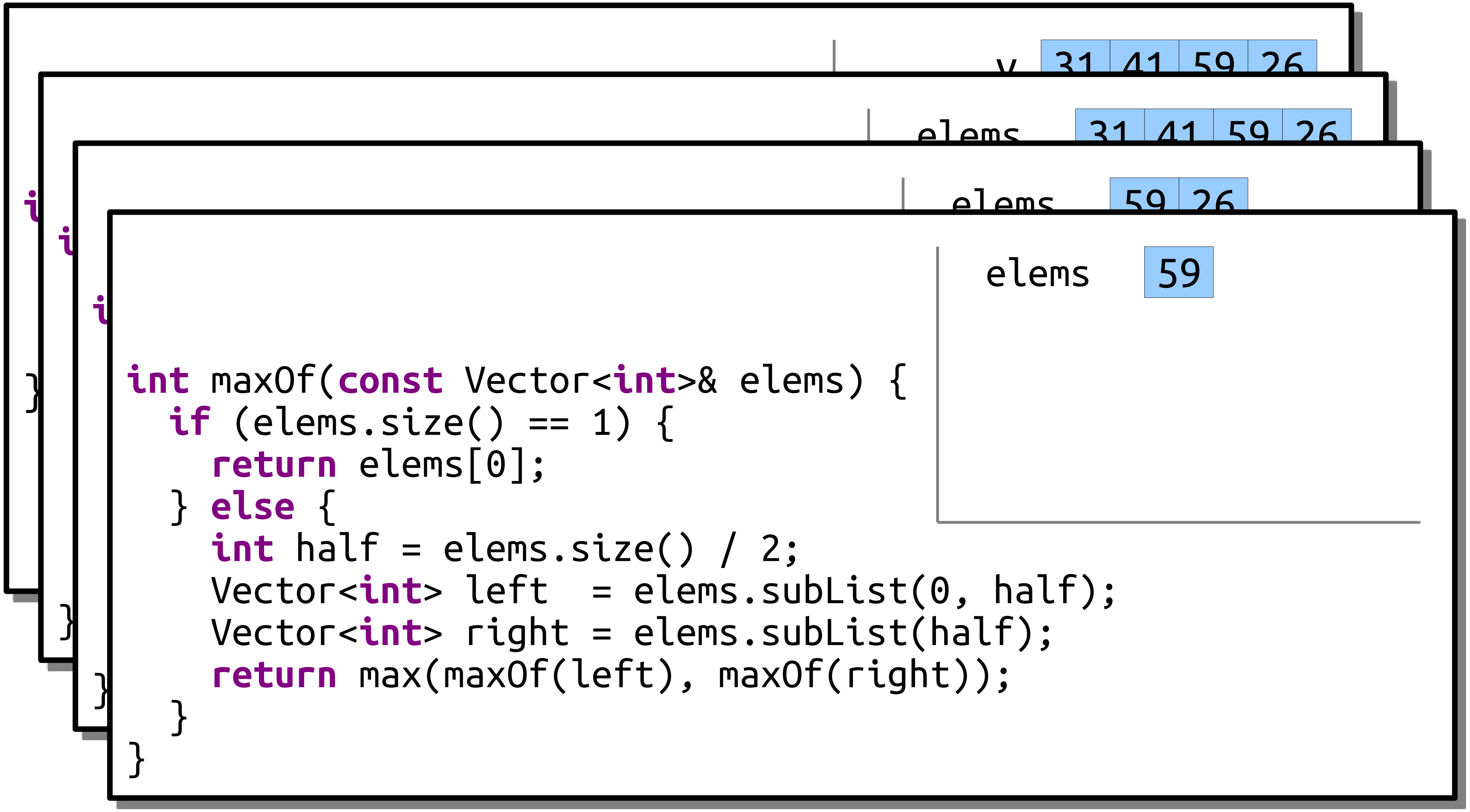
Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

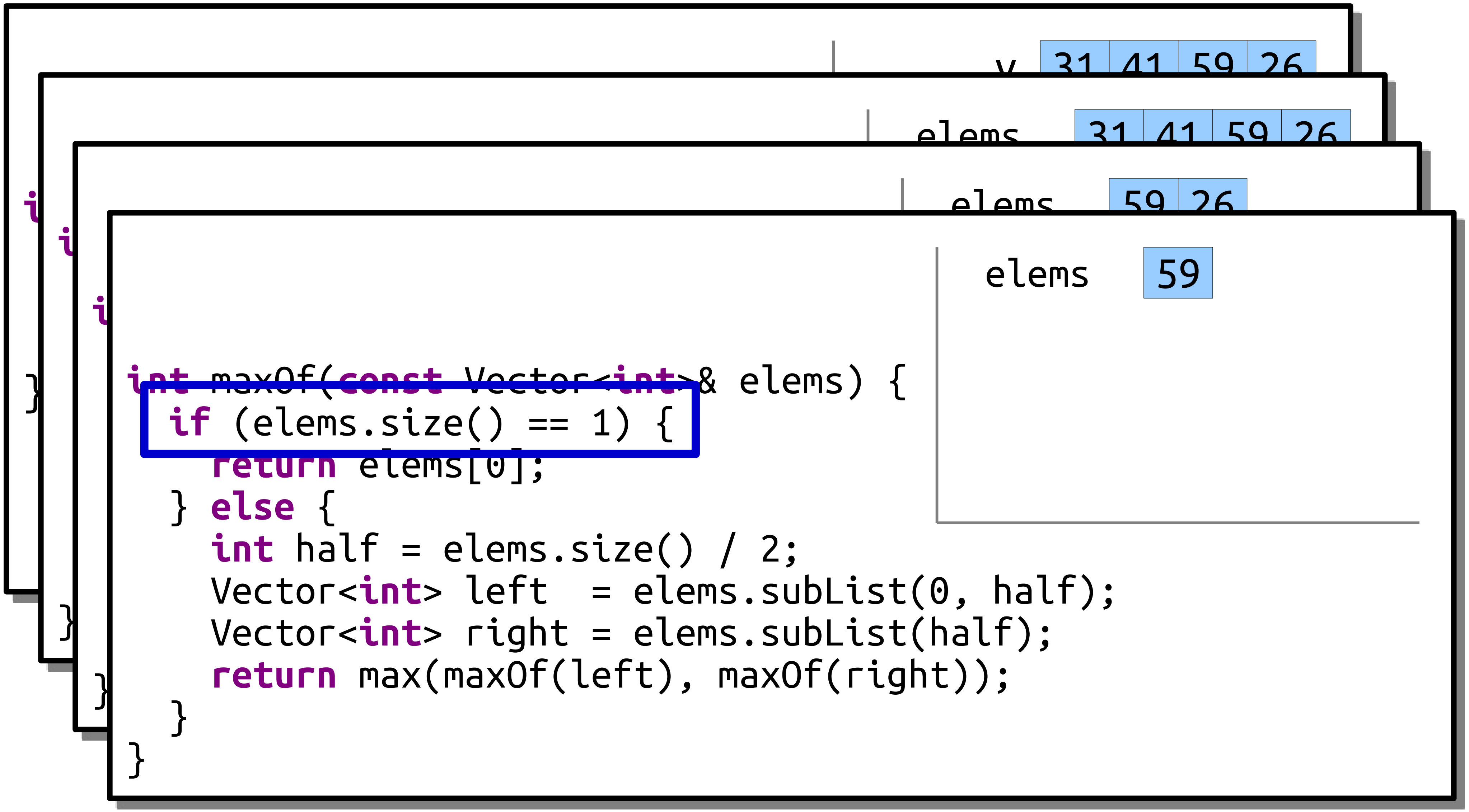
elems	59	26
half	1	
left	59	
right	26	

Tracing the Recursion



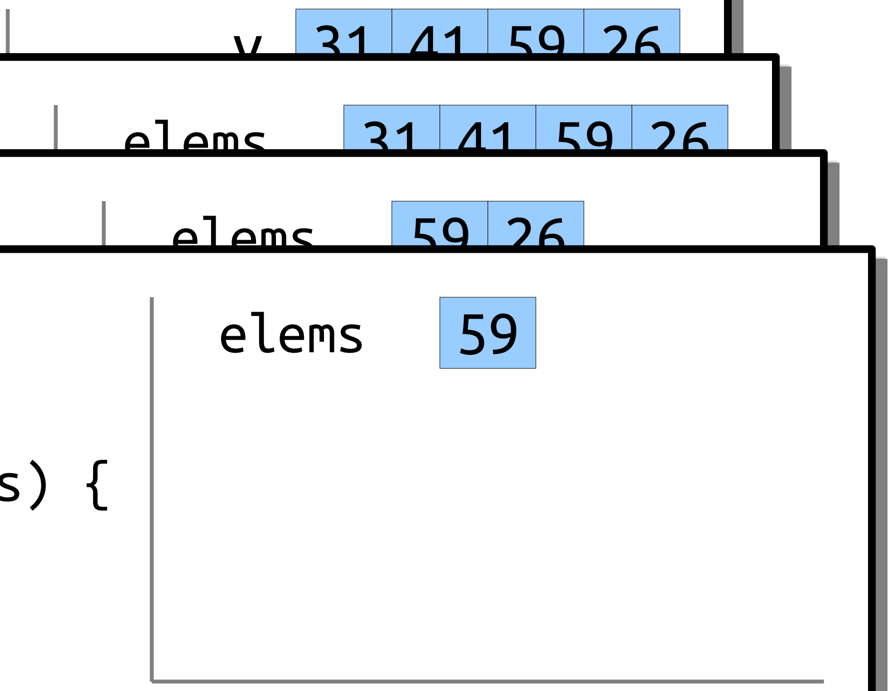
```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion

v 31 41 59 26

elems 31 41 59 26

elems 59 26

elems 59

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0]; 59
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion

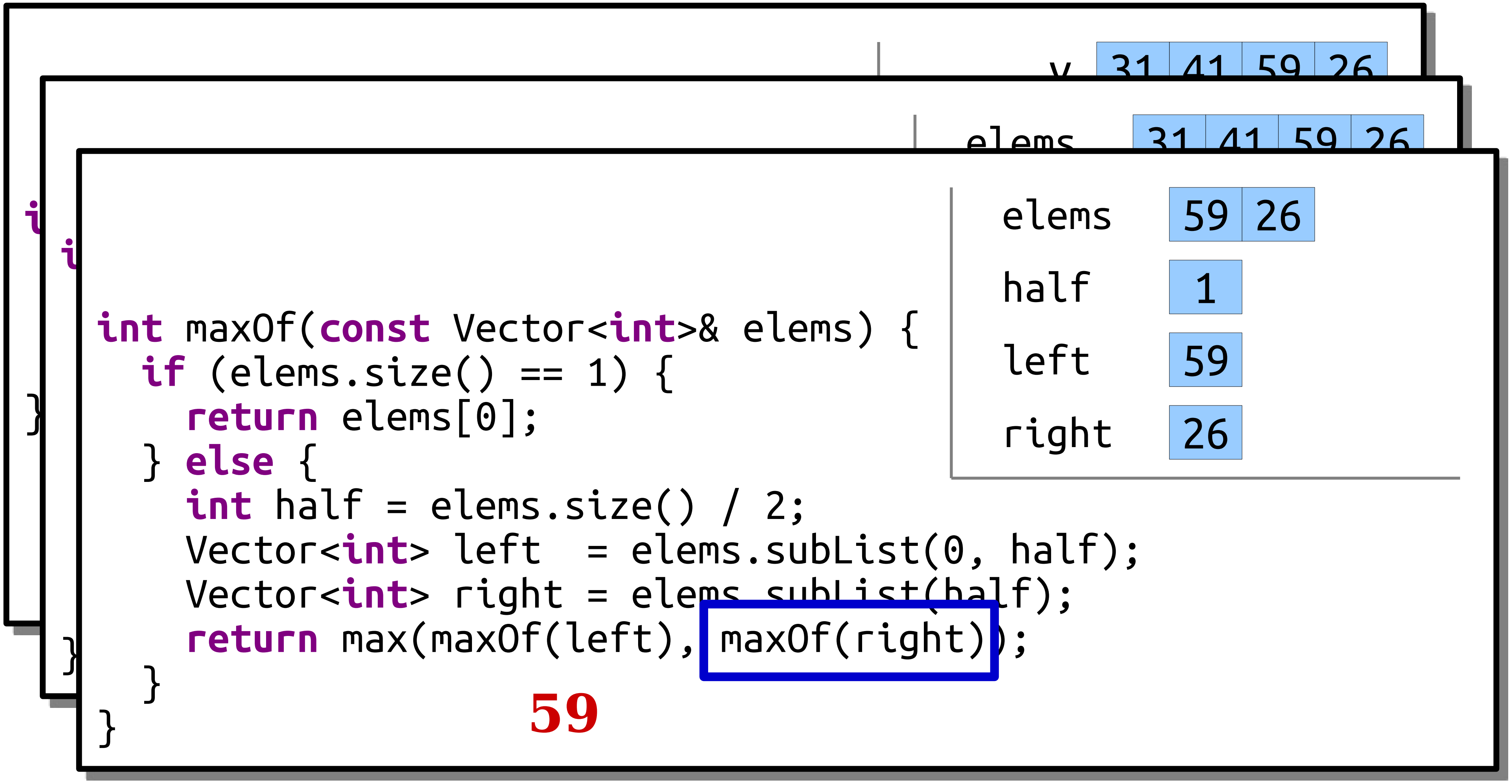
v	31	41	59	26
elems	31	41	59	26

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

59

elems	59	26
half	1	
left	59	
right	26	

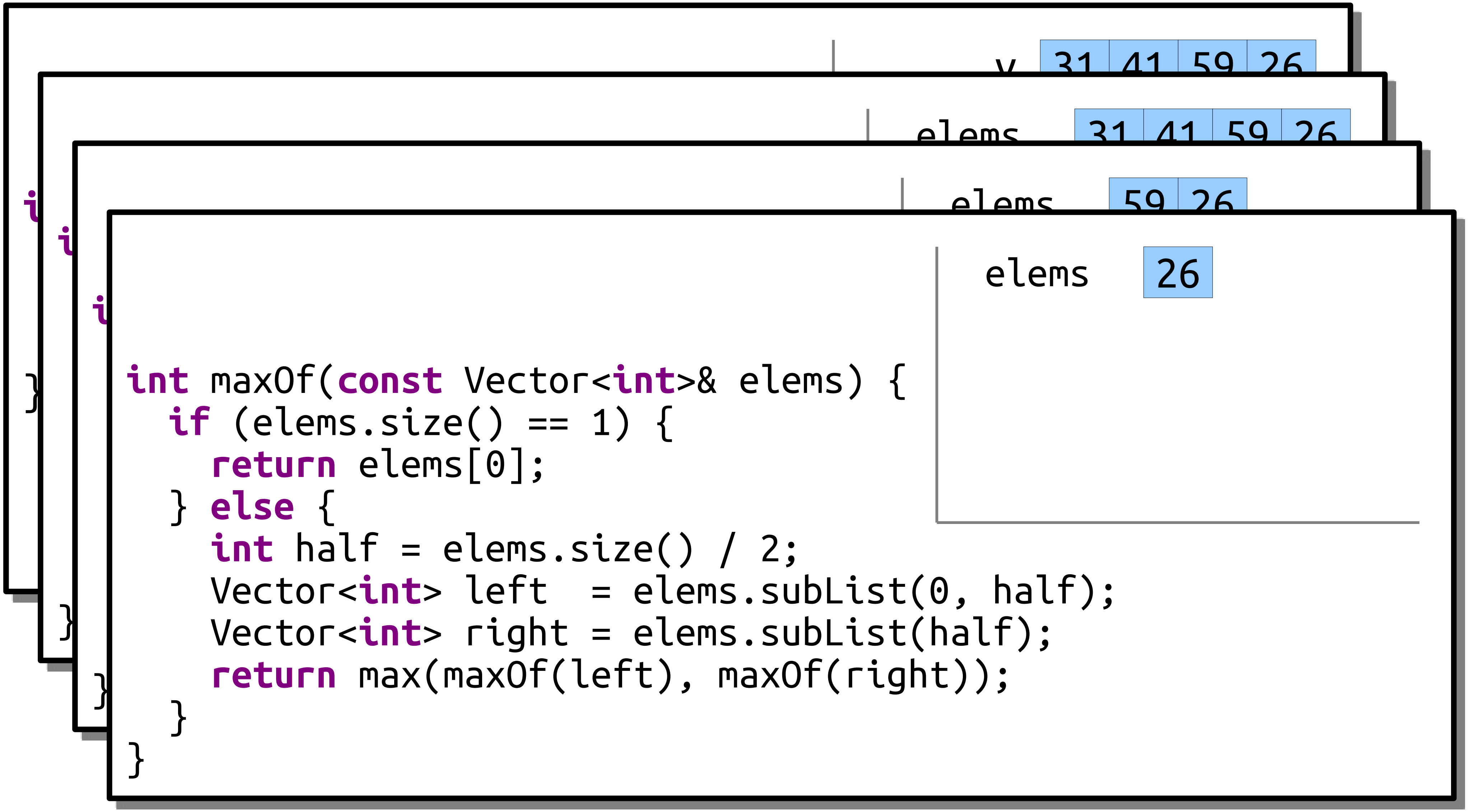
Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

elems	59	26
half	1	
left	59	
right	26	

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion

v | 31 | 41 | 59 | 26

elems | 31 | 41 | 59 | 26

elems | 59 | 26

elems | 26

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion

v 31 41 59 26

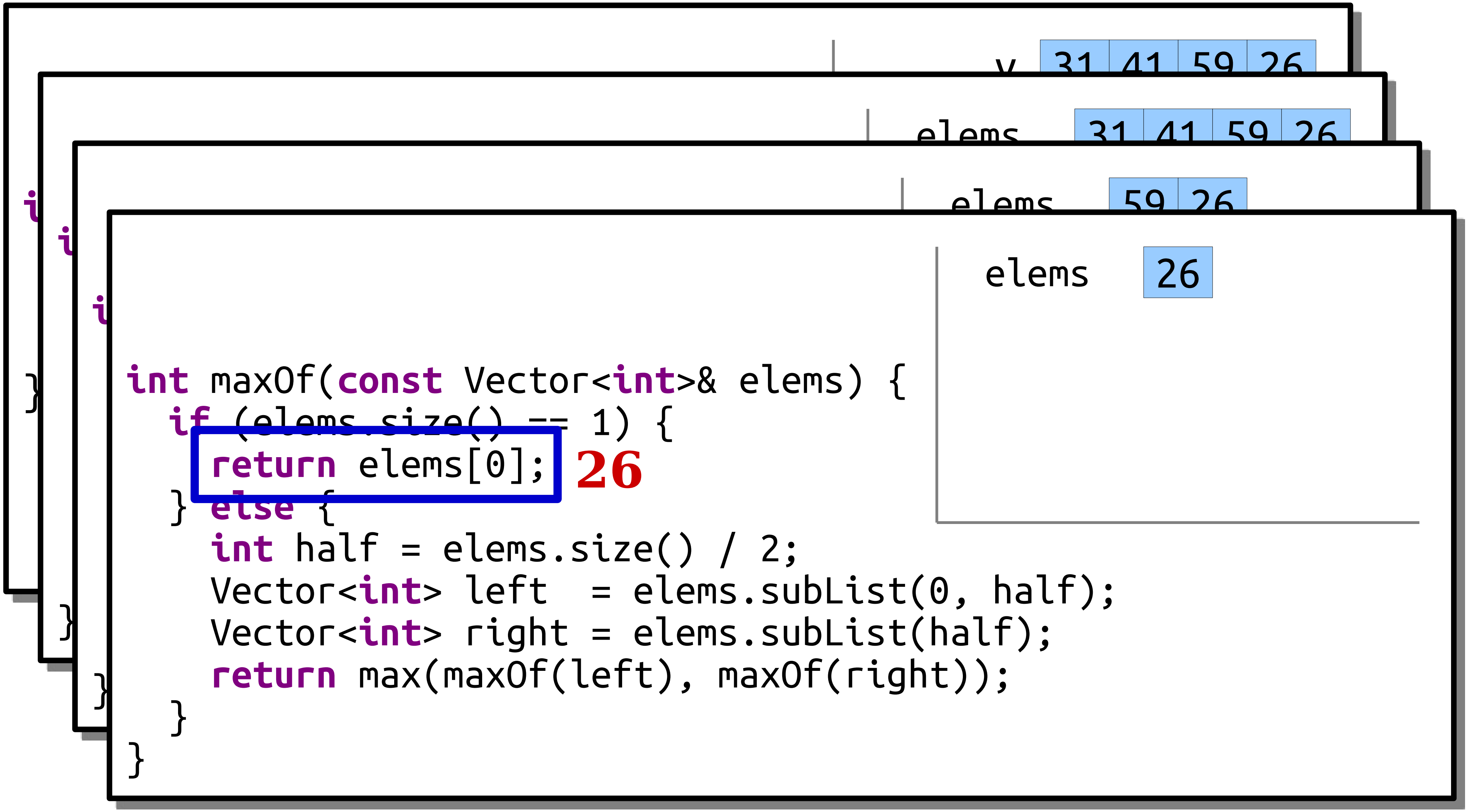
elems 31 41 59 26

elems 59 26

elems 26

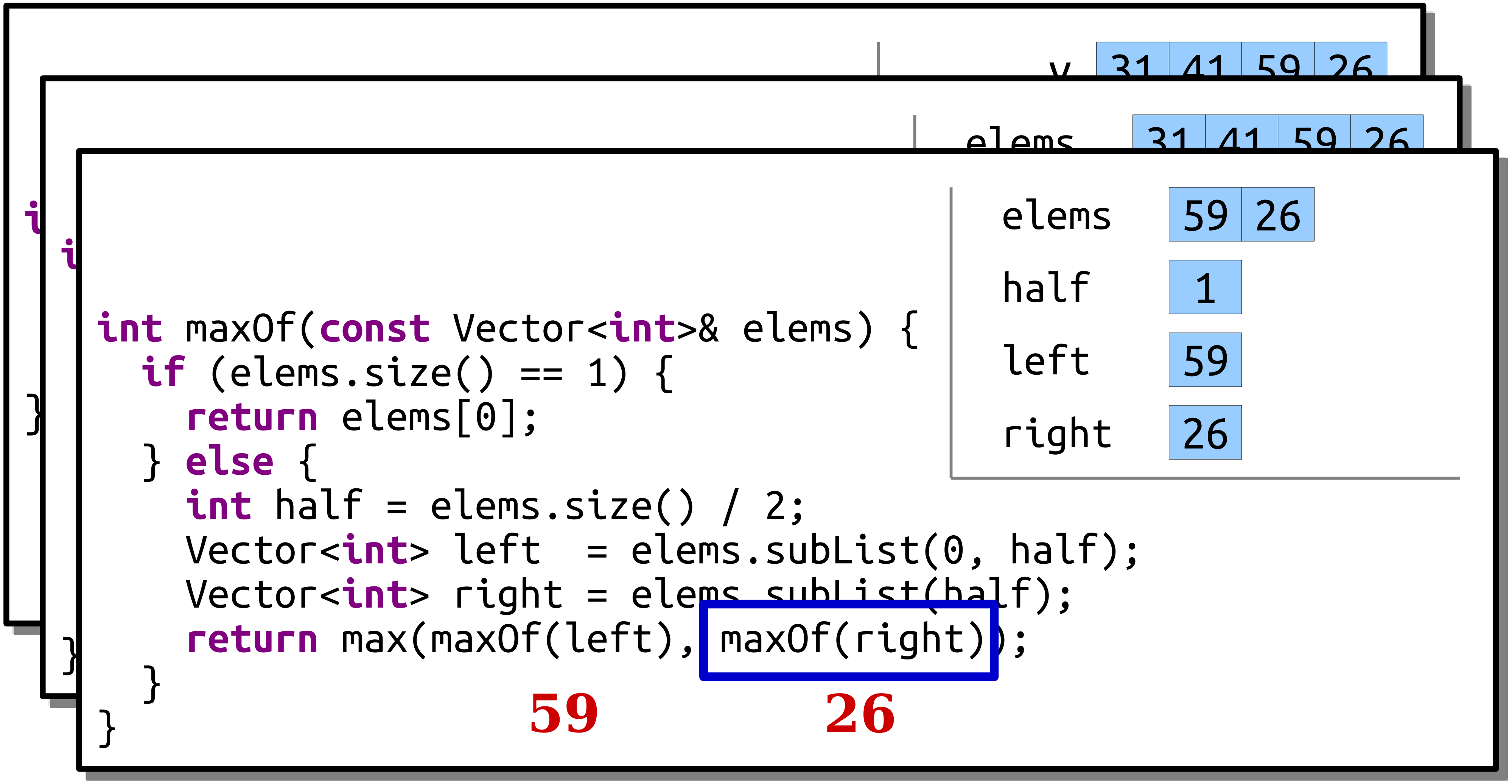
```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



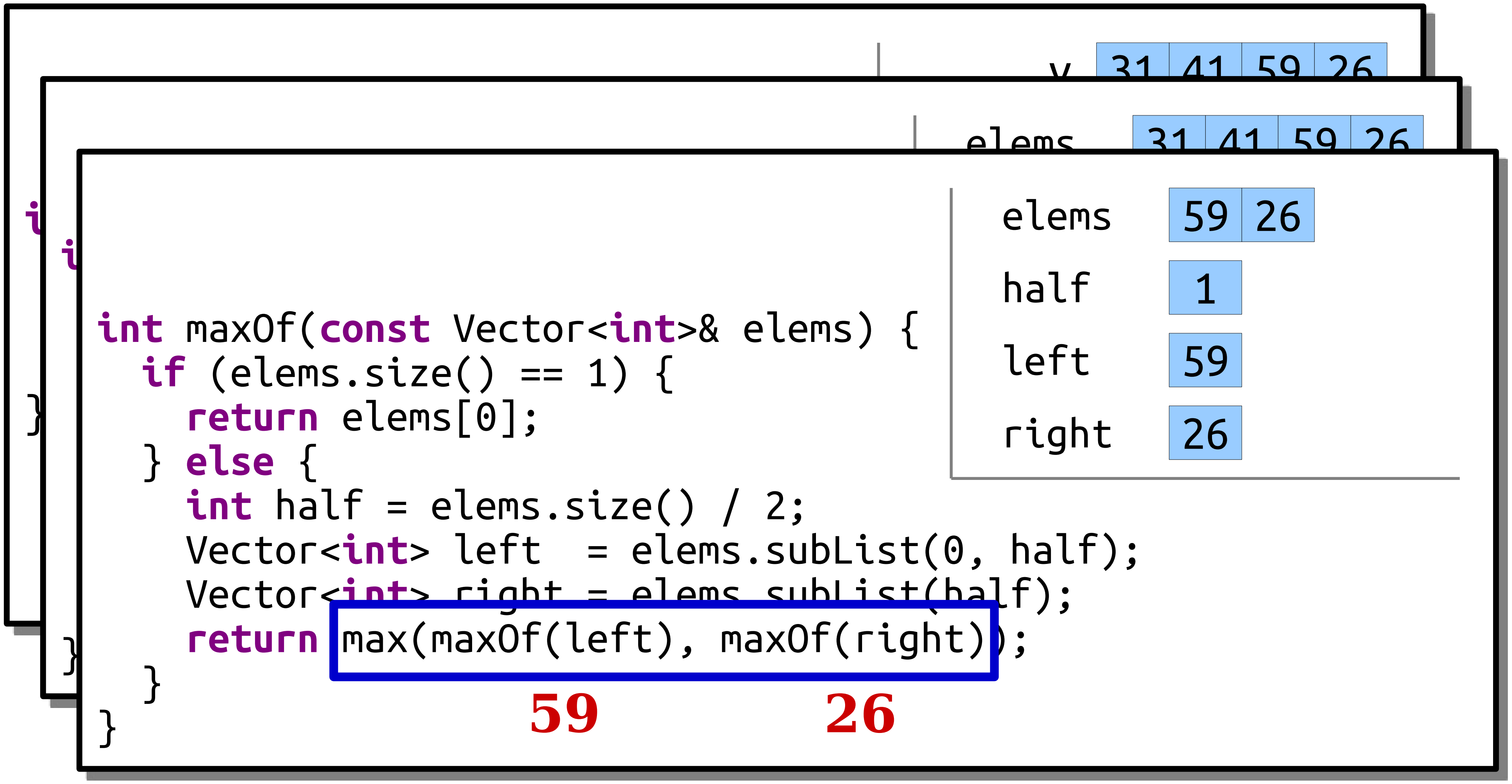
```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

Tracing the Recursion



```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

elems	59 | 26
half	1
left	59
right	26

Tracing the Recursion

v	31	41	59	26

elems	31	41	59	26

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

elems	59	26
half	1	
left	59	
right	26	

59

Tracing the Recursion

v 31 41 59 26

elems 31 41 59 26

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

59

elems	59 26
half	1
left	59
right	26

Tracing the Recursion

v	31	41	59	26

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

elems	31	41	59	26
half	2			
left	31	41		
right	59	26		

41 **59**

Tracing the Recursion

elems	31	41	59	26
half	2			
left	31	41		
right	59	26		

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

41 **59**

Tracing the Recursion

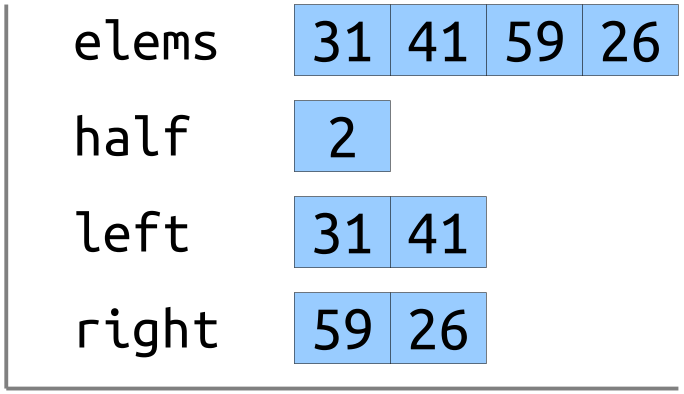
```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```

59

elems	31	41	59	26
half	2			
left	31	41		
right	59	26		

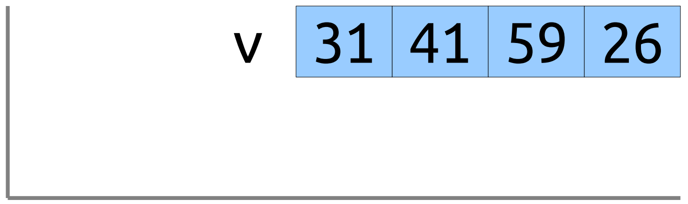
Tracing the Recursion

```
int maxOf(const Vector<int>& elems) {
  if (elems.size() == 1) {
    return elems[0];
  } else {
    int half = elems.size() / 2;
    Vector<int> left  = elems.subList(0, half);
    Vector<int> right = elems.subList(half);
    return max(maxOf(left), maxOf(right));
  }
}
```



59

Tracing the Recursion



```
int main() {
    Vector<int> v = { 31, 41, 59, 26 };
    cout << maxOf(v) << endl;
    return 0;
}
```

59